12. Find the intervals on which f is increasing or decreasing, and find the local maximum and minimum values of f.
$f(x) = x^{2/3}(x-3)$

Sol.
$f(x) = x^{2/3}(x-3) \Rightarrow f'(x) = \frac{2}{3}x^{-1/3}(x-3) + x^{2/3}(1) = \frac{1}{3}x^{-1/3}(2x-6+3x) = \frac{1}{3}x^{-1/3}(5x-6)$
Hence we may make the following table:

Interval	$x^{-1/3}$	$5x-6$	$f'(x)$	f
$x<0$	$-$	$-$	$+$	increasing on $(-\infty, 0)$
$0<x<\frac{6}{5}$	$+$	$-$	$-$	decreasing on $(0, \frac{6}{5})$
$x>\frac{6}{5}$	$+$	$+$	$+$	increasing on $(\frac{6}{5}, \infty)$

f changes from increasing to decreasing at $x=0$ and from decreasing to increasing at $x=\frac{6}{5}$. Thus, $f(0)=0$ is a local maximum value and $f(\frac{6}{5}) = (\frac{6}{5})^{2/3}(-\frac{9}{5}) \approx -2.03$ is a local minimum value.

14. Find the intervals on which f is increasing or decreasing, and find the local maximum and minimum values of f.
$f(x) = x + \frac{4}{x^2}$

Sol.
$f(x) = x + \frac{4}{x^2} \Rightarrow f'(x) = 1 - 8x^{-3} = \frac{x^3-8}{x^3} = \frac{(x-2)(x^2+2x+4)}{x^3}$
The factor (x^2+2x+4) is always positive and does not affect the sign of $f'(x)$.Hence we may make the following table:

Interval	x^3	$x-2$	$f'(x)$	f
$x<0$	$-$	$-$	$+$	increasing on $(-\infty, 0)$
$0<x<2$	$+$	$-$	$-$	decreasing on $(0, 2)$
$x>2$	$+$	$+$	$+$	increasing on $(2, \infty)$

$x=0$ is not in the domain of f. f changes from decreasing to increasing at $x=2$. Thus, $f(2)=3$ is a local minimum value.

16. Find the intervals on which f is increasing or decreasing, and find the local maximum and minimum values of f.
$f(x) = x^4e^{-x}$

Sol.
$f(x) = x^4e^{-x} \Rightarrow f'(x) = 4x^3e^{-x} + x^4(-e^{-x}) = x^3e^{-x}(4-x)$
Thus, $f'(x) > 0$ if $0<x<4$ and $f'(x)<0$ if $x<0$ or $x>4$.
So f is increasing on $(0,4)$ and decreasing on $(-\infty, 0)$ and $(4, \infty)$.
Hence $f(0)=0$ is local minimum value and $f(4) = 256e^-4 \approx 4.69$ is a local maximum value.

24. (a) Find the intervals on which f is increasing or decreasing.
(b) Find the local maximum and minimum values of f.
(c) Find the intervals of concavity and the inflection points.

$f(x) = \dfrac{x}{x^2+1}$

Sol.

(a)

$f(x) = \dfrac{x}{x^2+1} \Rightarrow f'(x) = \dfrac{(1)(x^2+1)-(x)(2x)}{(x^2+1)^2} = \dfrac{1-x^2}{(x^2+1)^2} = -\dfrac{(x+1)(x-1)}{(x^2+1)^2}$

Thus, $f'(x) > 0$ if $(x+1)(x-1) < 0 \iff -1 < x < 1$ and $f'(x) < 0$ if $x < -1$ or $x > 1$. So f is increasing on $(-1, 1)$ and f is decreasing on $(-\infty, -1)$ and $(1, \infty)$

(b)

f changes from decreasing to increasing at $x = -1$ and from increasing to decreasing at $x = 1$. Thus, $f(-1) = -\frac{1}{2}$ is a local minimum and $f(1) = \frac{1}{2}$ is a local maximum value.

(c)

$f''(x) = \dfrac{(-2x)((x^2+1)^2) - (1-x^2)(2(x^2+1)(2x))}{((x^2+1)^2)^2} = \dfrac{(x^2+1)(-2x)[(x^2+1)+2(1-x^2)]}{(x^2+1)^4} = \dfrac{2x(x^2-3)}{(x^2+1)^3}$

$f''(x) > 0 \iff -\sqrt{3} < x < 0$ or $x > \sqrt{3}$, and $f''(x) < 0 \iff x < -\sqrt{3}$ or $0 < x < \sqrt{3}$.
Thus, f is concave upward on $(-\sqrt{3}, 0)$ and $(\sqrt{3}, \infty)$ and concave downward on $(-\infty, -\sqrt{3})$ and $(0, \sqrt{3})$. There are inflection points at $(-\sqrt{3}, \frac{-\sqrt{3}}{4})$, $(0, 0)$, and $(\sqrt{3}, \frac{\sqrt{3}}{4})$.

28. (a) Find the intervals on which f is increasing or decreasing.
(b) Find the local maximum and minimum values of f.
(c) Find the intervals of concavity and the inflection points.
$f(x) = \cos^2 x - 2\sin x, \quad 0 \leq x \leq 2pi$

Sol.

(a)

$f(x) = \cos^2 x - 2\sin x \Rightarrow f'(x) = -2\cos x \sin x - 2\cos x = -2\cos x(1 + \sin x)$

Note that $1 + \sin x \geq 0$ [since $\sin x \geq -1$],
with equality $\iff \sin x = -1 \iff x = \frac{3\pi}{2}$[since $0 \leq x \leq 2\pi$] $\Rightarrow \cos x = 0$.
Thus, $f'(x) > 0 \iff \cos x < 0 \iff \frac{\pi}{2} < x < \frac{3\pi}{2}$ and $f'(x) < 0 \iff \cos x > 0 \iff 0 < x < \frac{\pi}{2}$ or $\frac{3\pi}{2} < x < 2\pi$. So f is increasing on $(\frac{\pi}{2}, \frac{3\pi}{2})$ and f is decreasing on $(0, \frac{\pi}{2})$ and $(\frac{3\pi}{2}, 2\pi)$

(b)

f changes from decreasing to increasing at $x = \frac{\pi}{2}$ and from increasing to decreasing at $x = \frac{3\pi}{2}$. Thus, $f(\frac{\pi}{2}) = -2$ is a local minimum and $f(\frac{3\pi}{2}) = 2$ is a local maximum value.

(c)

$$\begin{aligned} f''(x) &= 2\sin x(1+) - 2\cos^2 x = 2(\sin x + \sin^2 x - (1 - \sin^2 x)) \\ &= 2(2\sin^2 x + \sin x) = 2(2\sin x - 1)(\sin x + 1) \end{aligned}$$

so $f''(x) > 0 \iff \sin x > \dfrac{1}{2} \iff \dfrac{\pi}{6} < x < \dfrac{5\pi}{6}$, and
$f''(x) < 0 \iff \sin x < \dfrac{1}{2}$ and $\sin x \neq -1 \iff 0 < x < \frac{\pi}{6}$ or $\dfrac{5\pi}{6} < x < \dfrac{3\pi}{2}$ or $\dfrac{3\pi}{2} < x < 2\pi$.
Thus, f is concave upward on $(\frac{\pi}{6}, \frac{5\pi}{6})$ and concave downward on $(0, \frac{\pi}{6})$, $(\frac{5\pi}{6}, \frac{3\pi}{2})$, and $(\frac{3\pi}{2}, 2\pi)$. There are inflection points at $(\frac{\pi}{6}, \frac{-1}{4})$ and $(\frac{5\pi}{6}, \frac{-1}{4})$.

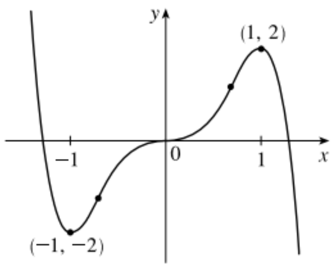
50. (a) Find the intervals of increase or decrease.
(b) Find the local maximum and minimum values.
(c) Find the intervals of concavity and the inflection points.
(d) Use the information from parts (a)-(c) to sketch the graph. You may want to check your work with a graphing calculator or computer.
$h(x) = 5x^3 - 3x^5$

Sol.
(a)
$h(x) = 5x^3 - 3x^5 \Rightarrow h'(x) = 15x^2 - 15x^4 = 15x^2(1+x)(1-x)$.
$h'(x) > 0 \iff -1 < x < 0$ and $0 < x < 1$ [note that $h'(0) = 0$] and $h'(x) < 0 \iff x < -1$ or $x > 1$.
So h is increasing on $(-1, 1)$ and h is decreasing on $(-\infty, -1)$ and $(1, \infty)$.

(b)
h changes from decreasing to increasing at $x = -1$, so $h(-1) = -2$ is a local minimum value. h changes from increasing to decreasing at $x = 1$, so $h(1) = 2$ is a local maximum value.

(c)
$h''(x) = 30x - 60x^3 = 30x(1 - 2x^2)$.
$h''(x) = 0 \iff x = 0$ or $x = \pm\frac{1}{\sqrt{2}}$.
$h''(x) > 0$ on $(-\infty, -\frac{1}{\sqrt{2}})$ and $(0, \frac{1}{\sqrt{2}})$, and $h''(x) < 0$ on $(-\frac{1}{\sqrt{2}}, 0)$ and $(\frac{1}{\sqrt{2}}, \infty)$
So h is concave upward on $(-\infty, -\frac{1}{\sqrt{2}})$ and $(0, \frac{1}{\sqrt{2}})$ and concave downward on $(-\frac{1}{\sqrt{2}}, 0)$ and $(\frac{1}{\sqrt{2}}, \infty)$.
There are inflection points at $(-\frac{1}{\sqrt{2}}, -\frac{7}{4\sqrt{2}})$, $(0, 0)$, and $(\frac{1}{\sqrt{2}}, \frac{7}{4\sqrt{2}})$

(d)



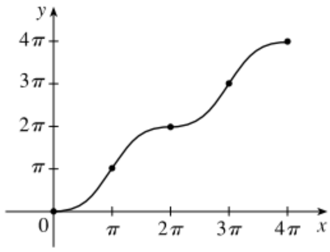
58. (a) Find the intervals of increase or decrease.
(b) Find the local maximum and minimum values.
(c) Find the intervals of concavity and the inflection points.
(d) Use the information from parts (a)-(c) to sketch the graph. You may want to check your work with a graphing calculator or computer.
$S(x) = x - \sin x, \quad 0 \le x \le 4\pi$

Sol.
(a)
$S(x) = x - \sin x \Rightarrow S'(x) = 1 - \cos x$.
$S'(x) = 0 \iff \cos x = 1 \iff x = 0,\ 2\pi,$ and 4π.
$S'(x) > 0 \iff \cos x < 1$, which is true for all x except integer multiples of 2π, so S is increasing on $(0, 4\pi)$ since $S'(2\pi) = 0$.

(b)
There is no local maximum or minimum.

(c)
$S''(x) = \sin x$.
$S''(x) > 0$ if $0 < x < \pi$ or $2\pi < x < 3\pi$, and
$S''(x) < 0$ if $\pi < x < 2\pi$ or $3\pi < x < 4\pi$, and
So S is concave upward on $(0, \pi)$ and $(2\pi, 3\pi)$ and concave downward on $(\pi, 2\pi)$ and $(3\pi, 4\pi)$.
There are inflection points at (π, π), $(2\pi, 2\pi)$, and $(23\pi, 3\pi)$

(d)



62. (a) Find the vertical and horizontal asymptotes.
(b) Find the intervals of increase or decrease.
(c) Find the local maximum and minimum values.
(d) Find the intervals of concavity and the inflection points.
(e) Use the information from parts (a)-(d) to sketch the graph.
$f(x) = \dfrac{e^x}{1-e^x}$

Sol.

Note that $f(x)$ has domain $\{x|1-e^x \neq 0\} = \{x|e^x \neq 1\} = \{x|x \neq 0\}$

(a)

$\lim\limits_{x\to\infty} \dfrac{e^x}{1-e^x} = \lim\limits_{x\to\infty} \dfrac{1}{1/e^x - 1} = \dfrac{1}{0-1} = -1$

So $y = -1$ is a horizontal asymptotes.

$\lim\limits_{x\to-\infty} \dfrac{e^x}{1-e^x} = \dfrac{0}{1-0} = 0$

So $y = 0$ is a horizontal asymptotes.

And $\lim\limits_{x\to0^-} \dfrac{e^x}{1-e^x} = \infty$

So $x = 0$ is a vertical asymptotes.

(b)

$f'(x) = \dfrac{e^x(1-e^x) - e^x(-e^x)}{(1-e^x)^2} = \dfrac{e^x(1-e^x+e^x)}{(1-e^x)^2} = \dfrac{e^x}{(1-e^x)^2}$.

$f'(x) > 0$ for $x \neq 0$, so f is increasing on $(-\infty, 0)$ and $(0, \infty)$.

(c)

There is no local maximum or minimum.

(d)

$f''(x) = \dfrac{e^x(1-e^x)^2 - e^x \cdot 2(1-e^x)(-e^x))}{((1-e^x)^2)^2} = \dfrac{e^x(1-e^x)(1-e^x+2e^x)}{(1-e^x)^4} = \dfrac{e^x(e^x+1)}{(1-e^x)^3}$.

$f''(x) > 0 \iff (1-e^x)^3 > 0 \iff e^x < 1 \iff x < 0$, and

$f''(x) < 0 \iff x > 0$

So f is concave upward on $(-\infty, 0)$ and concave downward on $(0, \infty)$.

There is no inflection point.

(e)