28. Differentiate the function.
$k(r) = e^r + r^e$

Sol.
$k'(r) = (e^r + r^e)' = (e^r)' + (r^e)' = e^r + er^{e-1}$

32. Differentiate the function.
$f(v) = \dfrac{\sqrt[3]{v} - 2ve^v}{v}$

Sol.
Note that $f(v) = \dfrac{\sqrt[3]{v} - 2ve^v}{v} = \dfrac{\sqrt[3]{v}}{v} - \dfrac{2ve^v}{v} = v^{-2/3} - 2e^v$.
So $f'(v) = -\dfrac{2}{3}v^{-5/3} - 2e^v$

37. Find an equation of the tangent line to the curve at the given point.
$y = 2x^3 - x^2 + 2, \quad (1, 3)$

Sol.
$y = 2x^3 - x^2 + 2 \Rightarrow y' = 6x^2 - 2x$.
At $(1, 3)$, $y' = 6(1)^2 - 2(1) = 4$ and the equation of the tangent line is
$y - 3 = 4(x - 1)$ or $y = 4x - 1$.

38. Find an equation of the tangent line to the curve at the given point.
$y = 2e^x + x, \quad (0, 2)$

Sol.
$y = 2e^x + x \Rightarrow y' = 2e^x + 1$.
At $(0, 2)$, $y' = 2e^0 + 1 = 3$ and the equation of the tangent line is
$y - 2 = 3(x - 0)$ or $y = 3x + 2$.

39. Find an equation of the tangent line to the curve at the given point.
$y = x + \dfrac{2}{x}, \quad (2, 3)$

Sol.
$y = x + \dfrac{2}{x} = x + 2x^{-1} \Rightarrow y' = 1 - 2x^{-2}$.
At $(2, 3)$, $y' = 1 - 2(2)^{-2} = \frac{1}{2}$ and the equation of the tangent line is
$y - 3 = \dfrac{1}{2}(x - 2)$ or $y = \dfrac{1}{2}x + 2$.

40. Find an equation of the tangent line to the curve at the given point.
$y = \sqrt[4]{x} - x, \quad (1, 0)$

Sol.
$y = \sqrt[4]{x} - x = x^{1/4} - x \Rightarrow y' = \dfrac{1}{4}x^{-3/4} - 1$.
At $(1, 0)$, $y' = \frac{1}{4}(1)^{-3/4} - 1 = -\frac{3}{4}$ and the equation of the tangent line is
$y - 0 = -\dfrac{3}{4}(x - 1)$ or $y = -\dfrac{3}{4}x + \dfrac{3}{4}$.

68. (a) Find equations of both lines through the point $(2,-3)$ that are tangent to the parabola $y = x^2 + x$.

(b) Show that there is no line through the point $(2,7)$ that is tangent to the parabola. Then draw a diagram to see why.

Sol.

(a)

If $y = x^2 + x$, then $y' = 2x + 1$. If the point at which a tangent meets the parabola is $(a, a^2 + a)$, then the slope of the tangent is $2a + 1$.

But since it passes through (2,-3), the slope must also be $\dfrac{\Delta y}{\Delta x} = \dfrac{a^2 + a + 3}{a - 2}$.

Therefore $2a + 1 = \dfrac{a^2 + a + 3}{a - 2}$. Solving this equation for a we get $a^2 + a + 3 = 2a^2 - 3a - 2 \iff a^2 - 4a - 5 = (a-5)(a+1) = 0 \iff a = 5$ or -1.

If $a = 5$, the point is $(5, 30)$ and the slope is 11, so the equation is $y - 30 = 11(x - 5)$ or $y = 11x - 25$.

If $a = -1$, the point is $(-1, 0)$ and the slope is -1, so the equation is $y - 0 = -1(x+1)$ or $y = -x - 1$.

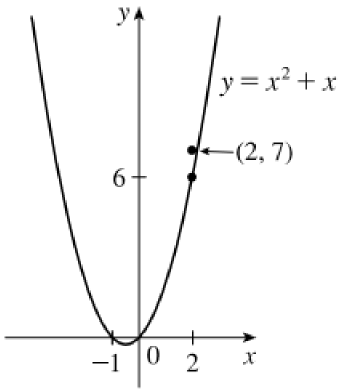
(b)

As in part (a), but using the point $(2, 7)$ we get the equation

$2a + 1 = \dfrac{a^2 + a + 7}{a - 2} \Rightarrow 2a^2 - 3a - 2 = a^2 + a - 7 \iff a^2 - 4a + 5 = 0.$

The last equation has no real solution, so there is no line through the point $(2, 7)$ that is tangent to the parabola.

The diagram shows that the point $(2, 7)$ is "inside" the parabola, but tangent lines to the parabola do not pass through points inside the parabola.



80. Suppose the curve $y = x^4 + ax^3 + bx^2 + cx + d$ has a tangent line when $x = 0$ with equation $y = 2x + 1$ and a tangent line when $x = 1$ with equation $y = 2 - 3x$. Find the values of a,b,c, and d.

Sol.

$y = x^4 + ax^3 + bx^2 + cx + d \Rightarrow y(0) = d$. Since the tangent line $y = 2x + 1$ is equal to 1 at $x = 0$, we must have $d = 1$.

$y' = 4x^3 + 3ax^2 + 2bx + c \Rightarrow y'(0) = c$. Since the slope of the tangent line at $x = 0$ is 2, we must have $c = 2$.

Now $y(1) = 1 + a + b + c + d = a + b + 4$ and the tangent line $y = 2 - 3x$ at $x = 1$ has y value of-1, we have $a + b + 4 = -1$. **(1)**

Also, $y'(1) = 4 + 3a + 2b + c = 3a + 2b + 6$ and the slope of the tangent line at $x = 1$ is -3, we have $3a + 2b + 6 = -3$. **(2)**

Combining **(1)** and **(2)**, we may solve for a and b and get $a = 1$ and $b = -6$.

Therefor the curve has equation $y = x^4 + x^3 - 6x^2 + 2x + 1$

86. Find numbers a and b such that the given function g is differentiable at 1.

$$g(x) = \begin{cases} ax^3 - 3x & \text{if } x \leqslant 1 \\ bx^2 + 2 & \text{if } x > 1 \end{cases}$$

Sol.
For $x < 1$, $g(x) = ax^3 - 3x \Rightarrow g'(x) = 3ax^2 - 3$. So $g'_-(1) = 3a - 3$.
For $x > 1$, $g(x) = bx^2 + 2 \Rightarrow g'(x) = 2bx$. So $g'_+(1) = 2b$.
For g to be differentiable at $x = 1$, we need $g'_-(1) = g'_+(1)$, so $3a - 3 = 2b$.
For g to be continuous (remember that differentiability implies continuity), we need $g_-(1) = a - 3 = b + 2 = g_+(1)$.
Solving the two equations we obtain $a = -7$ and $b = -12$.