10. Find dy/dx by the implicit differentiation.
$xe^x = x - y$

Sol.
$(xe^y)' = (x-y)' \Rightarrow e^y + xe^y(y') = 1 - y' \Rightarrow xe^y y' + y' = 1 - e^y \Rightarrow y' = \dfrac{1-e^y}{xe^y+1}$

12. Find dy/dx by the implicit differentiation.
$e^x \sin y = x + y$

Sol.
$(e^x \sin y)' = (x+y)' \Rightarrow e^x \sin y + e^x \cos y(y') = 1 + y' \Rightarrow e^x \cos y y' + y' = 1 - e^x \sin y \Rightarrow$
$y' = \dfrac{1 - e^x \sin y}{e^x \cos y + 1}$

14. Find dy/dx by the implicit differentiation.
$\tan(x-y) = 2xy^3 + 1$

Sol.
$(\tan(x-y))' = (2xy^3+1)' \Rightarrow \sec^2(x-y)\cdot(1-y') = 2y^3 + 2x(3y^2(y')) \Rightarrow$
$\sec^2(x-y) - \sec^2(x-y)y' = 2y^3 + 6xy^2y' \Rightarrow$
$y'(6xy^2 + \sec^2(x-y)) = \sec^2(x-y) - 2y^3 \Rightarrow y' = \dfrac{\sec^2(x-y) - 2y^3}{6xy^2 + \sec^2(x-y)}$

27. Use implicit differentiation to find an equation of the tangent line to the curve at the given point.
$ye^{\sin x} = x \cos y, \quad (0,0)$

Sol.
$(ye^{\sin x})' = (x\cos y)' \Rightarrow y'e^{\sin x} + ye^{\sin x}(\cos x) = \cos y - x \sin y(y') \Rightarrow$
$y'e^{\sin x} + x \sin y(y') = \cos y - ye^{\sin x}(\cos x) \Rightarrow y' = \dfrac{\cos y - ye^{\sin x}(\cos x)}{e^{\sin x} + x \sin y}$

When $x = 0$ and $y = 0$, we have $y' = \dfrac{\cos 0 - 0e^{\sin 0}(\cos 0)}{e^{\sin 0} + 0 \sin 0} = \dfrac{1-0}{1+0} = 1.$
So the equation is $y - 0 = 1(x - 0)$, or $y = x$

28. Use implicit differentiation to find an equation of the tangent line to the curve at the given point.
$\tan(x+y) + \sec(x-y) = 2, \quad (\pi/8, \pi/8)$

Sol.
$(\tan(x+y) + \sec(x-y))' = (2)' \Rightarrow \sec^2(x+y)(1+y') + \sec(x-y)\tan(x-y)(1-y') = 0 \Rightarrow$
$\sec^2(x+y) + \sec(x-y)\tan(x-y) = y'\sec(x-y)\tan(x-y) - y'\sec^2(x+y) = y'(\sec(x-y)\tan(x-y) - \sec^2(x+y)) \Rightarrow$
$y' = \dfrac{\sec(x-y)\tan(x-y) + \sec^2(x+y)}{\sec(x-y)\tan(x-y) - \sec^2(x+y)}$
When $x = \frac{\pi}{8}$ and $y = \frac{\pi}{8}$, $x - y = 0$ and $x + y = \frac{\pi}{4}$ and thus
$y' = \dfrac{\sec 0 \tan 0 + \sec^2 \frac{\pi}{4}}{\sec 0 \tan 0 - \sec^2 \frac{\pi}{4}} = \dfrac{0 + (\sqrt{2})^2}{0 - (\sqrt{2})^2} = -1$
So the equation is $y - \dfrac{\pi}{8} = -(x - \dfrac{\pi}{8})$, or $y = -x + \dfrac{\pi}{4}$

33. Use implicit differentiation to find an equation of the tangent line to the curve at the given point.
$x^2 + y^2 = (2x^2 + 2y^2 - x)^2, \quad (0, \frac{1}{2})$

Sol.
$(x^2 + y^2)' = ((2x^2 + 2y^2 - x)^2)' \Rightarrow 2x + 2yy' = 2(2x^2 + 2y^2 - x)(4x + 4yy' - 1)$
When $x = 0$ and $y = \frac{1}{2}$, we have $0 + y' = 2(0 + \dfrac{1}{2} - 0)(0 + 2y' - 1) \Rightarrow y' = 2y' - 1 \Rightarrow y' = 1.$
So the equation is $y - \frac{1}{2} = 1(x - 0)$ or $y = x + \frac{1}{2}$

34. Use implicit differentiation to find an equation of the tangent line to the curve at the given point.
$x^2y^2 = (y + 1)^2(4 - y^2), \quad (2\sqrt{3}, 1)$

Sol.
$(x^2y^2)' = ((y + 1)^2(4 - y^2))' \Rightarrow 2xy^2 + x^2(2yy') = (2(y + 1)y')(4 - y^2) + (y + 1)^2(-2yy') \Rightarrow$
$2xy^2 + 2x^2yy' = -2yy'(y + 1)^2 + 2y'(4 - y^2)(y + 1)$
When $x = 2\sqrt{3}$ and $y = 1$, we have $2(2\sqrt{3})1^2 + 2(2\sqrt{3})^2 \cdot 1 \cdot y' = -2 \cdot 1 \cdot y'(1+1)^2 + 2y'(4-1^2)(1+1) \Rightarrow$
$4\sqrt{3} + 24y' = -8y' + 12y' = 4y' \Rightarrow y' = \dfrac{-\sqrt{3}}{5}$
So the equation is $y - 1 = \frac{-\sqrt{3}}{5}(x - 2\sqrt{3})$ or $y = \frac{-\sqrt{3}}{5}x + \frac{11}{5}$

42. Find y'' by implicit differentiation.
$x^3 - y^3 = 7$

Sol.
$x^3 - y^3 = 7 \Rightarrow 3x^2 - 3y^2y' = 0 \Rightarrow y' = \dfrac{x^2}{y^2} \Rightarrow$
$$y'' = \frac{(2x)(y^2) - (x^2)(2yy')}{(y^2)^2} = \frac{2xy^2 - 2x^2y \cdot \frac{x^2}{y^2}}{y^4} = \frac{2xy^2 - 2\frac{x^4}{y}}{y^4} = \frac{2xy^3 - 2x^4}{y^5}$$
You may further simplify it by $y'' = \dfrac{2xy^3 - 2x^4}{y^5} = \dfrac{2x(y^3 - x^3)}{y^5} = \dfrac{-14x}{y^5}$

50. Show that the sum of the $x-$ and $y-$intercepts of any tangent line to the curve $\sqrt{x} + \sqrt{y} = \sqrt{c}$ is equal to c.

Sol.
$\sqrt{x} + \sqrt{y} = \sqrt{c} \Rightarrow \dfrac{1}{2\sqrt{x}} + \dfrac{y'}{2\sqrt{y}} = 0 \Rightarrow y' = -\dfrac{\sqrt{y}}{\sqrt{x}}.$
Hence the equation of the tangent line at a given point (x_0, y_0) is $y - y_0 = -\frac{\sqrt{y_0}}{\sqrt{x_0}}(x - x_0)$
Set $x = 0$ for $y-$intercepts, we get $y - y_0 = -\frac{\sqrt{y_0}}{\sqrt{x_0}}(0 - x_0) = \sqrt{x_0}\sqrt{y_0} \Rightarrow y = y_0\sqrt{x_0}\sqrt{y_0}.$
Set $y = 0$ for $x-$intercepts, we get $0 - y_0 = -\frac{\sqrt{y_0}}{\sqrt{x_0}}(x - x_0) \Rightarrow x = x_0 + y_0\frac{\sqrt{x_0}}{\sqrt{y_0}} = x_0 + \sqrt{x_0}\sqrt{y_0}.$
Hence the sum of them is:
$(y_0 + \sqrt{x_0}\sqrt{y_0}) + (x_0 + \sqrt{x_0}\sqrt{y_0}) = (\sqrt{x_0})^2 + 2\sqrt{x_0}\sqrt{y_0} + (\sqrt{y_0})^2 = (\sqrt{x_0} + \sqrt{y_0})^2 = (\sqrt{c})^2 = c$

64. Find equations of both the tangent lines to the ellipse $x^2 + 4y^2 = 36$ that pass through the point $(12, 3)$.

Sol.

$x^2 + 4y^2 = 36 \Rightarrow 2x + 8yy' = 0 \Rightarrow y' = -\frac{x}{4y}$.
Let (a, b) be a point on the ellipse whose tangent line passes through $(12, 3)$.
The tangent line is then $y - 3 = -\dfrac{a}{4b}(x - 12)$, so $b - 3 = -\dfrac{a}{4b}(a - 12) \Rightarrow 4b^2 - 12b = -a^2 + 12a$
$\Rightarrow 12(a + b) = 4b^2 + a^2 = 36 \Rightarrow a + b = 3$.
Substituting $b = 3 - a$ into the equation $a^2 + 4b^2 = 36$ gives:
$a^2 + 4(3 - a)^2 = a^2 + 4a^2 - 24a + 36 = 36 \Rightarrow 5a^2 - 24a = 0 \Rightarrow \ a = \dfrac{24}{5}$ or $a = 0$
Thus we obtain two points $(\frac{24}{5}, \frac{-9}{5})$ and $(0, 3)$ with slopes $\frac{2}{3}$ and 0, respectively.
So the equations of the two tangent lines are
$y - \dfrac{-9}{5}) = \dfrac{2}{3}(x - \dfrac{24}{5})$ or $y = \dfrac{2}{3} - 5$
$y - 3 = 0(x - 0)$ or $y = 3$

A graph of the ellipse and the tangent lines confirms our results.