微積分1–4.9 Antiderivatives-Video 2

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Plan of the this Video

In this video, we will cover
materials in 4.9 Antiderivatives. In this video, we will look the indefinite integral of some known functions and examples. We also learn how to graph antiderivative of a function.

If we know the derivative of a function is $g(x)$ then we can determine its anti-derivative.
A basic indefinite integral is one that can be computed either by recognizing the integrand as the derivative of a familiar function or by reversing the Power Rule for Derivatives. Recall $\int f(x)dx = F(x) + C$ if $F'(x) = f(x)$. To check this is true, we need to verify that $F'(x) = f(x)$.

Theorem

We know that $\frac{d}{dx}\frac{x^{a+1}}{a+1} = x^a$ if $a \neq -1$.
Thus $\int x^a dx = \frac{x^{a+1}}{a+1} + C$ if $a \neq -1$

We know that $\frac{d}{dx}\ln|x| = \frac{1}{x}$, .
Thus $\int \frac{1}{x}dx = \ln|x| + C$.

We know that $\frac{d}{dx}\sin(x) = \cos(x)$.
Thus $\int \cos(x)dx = \sin(x) + C$.

We know that $\frac{d}{dx}[-\cos(x)] = \sin(x)$
Thus $\int \sin(x)dx = -\cos(x) + C$.

We know that $\frac{d}{dx}[\tan(x)] = \sec^2(x)$.
Thus $\int \sec^2(x)dx = \tan(x) + C$.

We know that $\frac{d}{dx}[-\cot(x)] = \csc^2(x)$.
Thus $\int \csc^2(x)dx = -\cot(x) + C$.

We know that $\frac{d}{dx}[\sec(x)] = \sec(x)\tan(x)$.
Thus $\int \sec(x)\tan(x)dx = \sec(x) + C$.

We know that $\frac{d}{dx}[-\csc(x)] = \csc(x)cot(x)$.
Thus $\int \csc(x)\cot(x)dx = -\csc(x) + C$.

We know that $\frac{d}{dx}e^x = e^x$.
Thus $\int e^x dx = e^x + C$.

We know that $\frac{d}{dx}\ln|x| = \frac{1}{x}$.
Thus $\int \frac{1}{x}dx = \ln|x| + C$.

Let $a > 0$ We know that $\frac{d}{dx}\frac{a^x}{\ln a} = a^x$.
Thus $\int a^x dx = \frac{a^x}{\ln a} + C$.

We know that $\frac{d}{dx}\arcsin(x) = \frac{1}{\sqrt{1-x^2}}$.
Thus $\int \frac{1}{\sqrt{1-x^2}}dx = \arcsin(x) + C$.

We know that $\frac{d}{dx}\arctan(x) = \frac{1}{1+x^2}$.
Thus $\int \frac{1}{1+x^2}dx = \arctan(x) + C$.

We know that $\frac{d}{dx}\text{arcsec}(x) = \frac{1}{x\sqrt{x^2-1}}$.
Thus $\int \frac{1}{x\sqrt{x^2-1}}dx = \text{arcsec}(x) + C$.

Antidifferentiation or indefinite integral Formulas

Suppose the antiderivative of $f(x)$ is $F(x)$ then $cF(x)$ is also an antiderivative of $cf(x)$. Thus $\int cf(x)dx = c\int f(x)dx$

Suppose the antiderivative of $f(x)$ and $g(x)$ are $F(x)$ and $G(x)$ then $F(x) + G(x)$ is also an antiderivative of $f(x) + g(x)$. Thus $\int f(x) + g(x)dx = \int f(x)dx + \int g(x)dx$.

So if the antiderivative of $f(x)$ and $g(x)$ are $F(x)$ and $G(x)$ then $\alpha F(x) + \beta G(x)$ is also an antiderivative of $\alpha f(x) + \beta g(x)$. $\int \alpha f(x) + \beta g(x)dx = \alpha\int f(x)dx + \beta\int g(x)dx$.

Example: 1. Find all functions g such that $g'(x) = x^5 - 4x^2 - 6$.
Solution:
$g(x) = \int x^5 - 4x^2 - 6dx = \frac{x^6}{6} - 4 \cdot \frac{x^3}{3} - 6x + C = \frac{x^6}{6} - \frac{4x^3}{3} - 6x + C$.
Remark: The antiderivative of a polynomial
$p(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$ is
$\frac{a_n x^{n+1}}{n+1} + \frac{a_{n-1} x^n}{n} + \cdots + \frac{a_1 x^2}{2} + a_0 x + C$ or
$\int a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0 dx =$
$\frac{a_n x^{n+1}}{n+1} + \frac{a_{n-1} x^n}{n} + \cdots + \frac{a_1 x^2}{2} + a_0 x + C$.

2. Find $\int \frac{x^5 - 3x^2 - 6}{x^3} dx$.
Solution: First, we simplify the expression
$\frac{x^5 - 3x^2 - 6}{x^3} = x^2 - 3x^{-1} - 6x^{-3}$. $\int x^2 - 3x^{-1} - 6x^{-3} dx =$
$\frac{x^3}{3} - 3 \ln |x| - 6 \frac{x^{-2}}{-2} + C = \frac{x^3}{3} - 3 \ln |x| + 3x^{-2} + C$.

3. Find $\int 4\sin(x) - \frac{x(x+1)}{2} + \frac{2x^3-5}{\sqrt{x}} dx$

Solution: First, we simplify the expression

$-\frac{x(x+1)}{2} + \frac{2x^3-5}{\sqrt{x}} = -\frac{x^2}{2} - \frac{x}{2} + \frac{2x^3-5}{x^{\frac{1}{2}}} = -\frac{x^2}{2} - \frac{x}{2} + 2x^{\frac{5}{2}} - 5x^{-\frac{1}{2}}$.

$$\int 4\sin(x) - \frac{x(x+1)}{2} + \frac{2x^3-5}{\sqrt{x}} dx$$

$$= \int 4\sin(x) - \frac{x^2}{2} - \frac{x}{2} + 2x^{\frac{5}{2}} - 5x^{-\frac{1}{2}} dx$$

$$= 4(-\cos(x)) - \frac{x^3}{2\cdot 3}. - \frac{x^2}{2\cdot 2} + 2\frac{x^{\frac{7}{2}}}{\frac{7}{2}} - 5\frac{x^{\frac{1}{2}}}{\frac{1}{2}} + C$$

$$= -4\cos(x) - \frac{x^3}{6}. - \frac{x^2}{4} + \frac{4}{7}x^{\frac{7}{2}} - 10x^{\frac{1}{2}} + C$$