微積分1 MATH 4006-15
Homework 3 Due Thursday during TA session , October 14, 2021
or Submit PDF files through NTU COOL by 5 pm if you are not in Taiwan or in quarantine.

You should show the detail of your works to get (partial) credits.

There is a quiz on October 14 , 5:30pm-6:20 pm during TA session. It be held on campus. Topics:2.1-2.8.

(1) (10 pts) Let $f(1) = 1$, $f'(1) = 0$, $g(0) = 0$ and $g'(0) = 1$. Find

$$\lim_{x \to \frac{\pi}{2}} \frac{f(\sin x)g(\cos x)}{x^2 - \frac{\pi}{2}x}.$$

(Hint: Let $h(x) = f(\sin x)g(\cos x)$ and find $h'(\frac{\pi}{2})$ by definition and product rule.)

(2) (10 pts) Suppose $f'(x)$ and $f''(x)$ exist. and $f(x)$ satisfies $x^2 + xf(x) + (f(x))^2 = k$ where k is a constant. Determine a and k if $f'(a) = f''(a) = 1$. Hint: Differentiate the equation $x^2 + xf(x) + (f(x))^2 = k$ twice and evaluate at a.

(3) (10 pts) Let

$$f(x) = \begin{cases} ae^x + bx, & x < 0; \\ m, & x = 0 \\ e^{-\frac{1}{x^2}} + \sqrt{x+4}, & 0 < x \end{cases}$$

Find conditions of m, a and b such that $f(x)$ is differentiable everywhere. Hint: Use the fact that $e^x \geq 1 + x$ for all x (i.e. the graph of $y = e^x$ is above the tangent to $y = e^x$ thru $(0,1)$).

(4) (a) (10 pts) Let $f(x) = e^{\tan(x)}$. Compute $f'(x)$ and $f''(x)$.
(b) (10 pts) Let $f(x) = x^{3^x} + 3^{x^3}$. Compute $f'(x)$
(c) (10 pts) Let $f(x) = \sqrt[3]{\sec x + \tan x}$. Compute $f'(x)$
(d) (10 pts) Let $f(x) = \frac{(e^{2x}+1)^3 \cot(2x)}{e^{(x^3+1)} + \csc(3x)}$, Compute $f'(x)$. You don't have to simplify your answer.
(e) (10 pts) Compute $\frac{d^3}{dx^3}\left(\frac{x}{\sqrt[3]{1+x}}\right)$.

(5) Let c, d be constants and $f(x) = ce^{ax}\sin(bx) + de^{ax}\cos(bx)$ where a, b are real and nonzero numbers.
(a) (10 pts) Compute $f'(x)$ and $f''(x)$.
(b) (10 pts) Find α and β such that $f''(x) + \alpha f'(x) + \beta f(x) = 0$.

Webwork homework `http://webwork.math.ntu.edu.tw/webwork2`
Do the one $1101MATH4006\_C13 - C16$
登入帳號密碼預設爲學號(英文大寫)，進去後可修改密碼，如有登入問題請用NTU信箱寄信至 ac1@math.ntu.edu.tw (來信請註明選到的班級及學號)

Orientation, due 10/31/2021 at 11:59pm (You should do this to understand the webwork system).
3.1 due 10/19/2021 at 11:59pm
3.2 due 10/19/2021 at 11:59pm
3.3 due 10/19/2021 at 11:59pm
3.4 due 10/19/2021 at 11:59pm

Practice problems (you don't have to submit this part.)
You can also do the following problems from the book for practice. But you do not have to turn in these problems. The solution of these problems will be posted in one or two weeks later. Solution will be posted at `http://www.math.ntu.edu.tw/~calc/cp_n_34461.html`

3.1 Derivatives of Polynomials and Exponential Functions 86, 88, 90
3.2 The Product and Quotient Rules 44, 50, 58
3.3 Derivatives of Trigonometric Functions 46, 62, 66
3.4 The Chain Rule 46, 48, 62, 69, 99