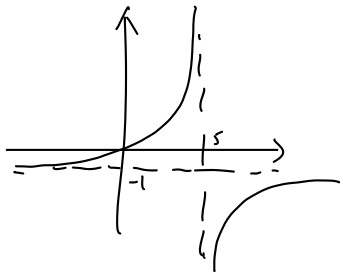
10. $y = \frac{x^2+5x}{25-x^2} = \frac{x}{5-x}$



V.A. : $x = 5$

H.A. : $\lim_{x\to\infty} y = -1$

$\lim_{x\to-\infty} y = -1$

$y' = \frac{5-x+x}{(5-x)^2} = \frac{5}{(5-x)^2} = 0$, no root → 都正 ↗

$y'' = \frac{10}{(5-x)^3}$, no root → 5前 +, 5後 −

∪ ∩

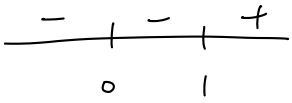
22. $y = (x-4)x^{\frac{1}{3}}$

V.A. : not exists singular pt.

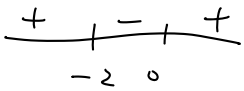
H.A. : $\lim_{x\to\infty} y \to \infty$ no H.A. exists

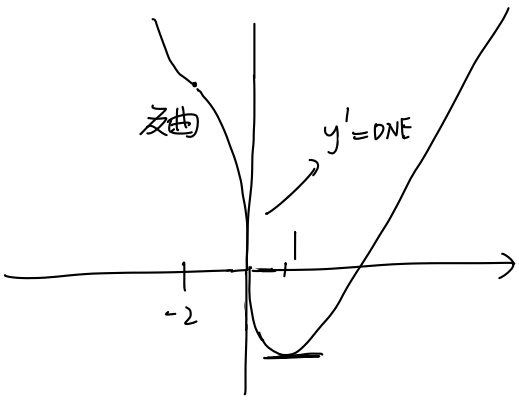
$\lim_{x\to-\infty} y \to \infty$

$y' = \frac{4(x-1)}{3x^{\frac{2}{3}}}$



$y'' = \frac{1}{9}x^{-\frac{5}{3}}(4x+8) = \frac{4}{9}x^{-\frac{5}{3}}(x+2)$





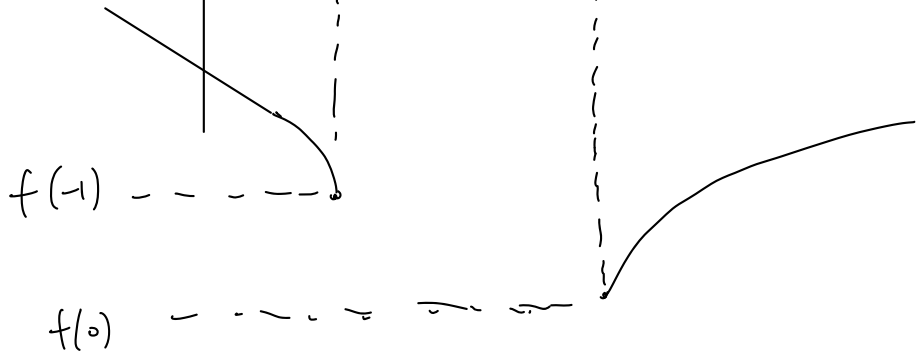
24.

$y = \sqrt{x^2+x} - x \longrightarrow$ 在 $x \in (0,-1)$ $y \notin \mathbb{R}$. 無法作圖

$y' = \frac{1}{2}\frac{2x+1}{\sqrt{x^2+x}} - 1$ ← $x = -1$ or 0 singular pt.

$y'' = \frac{1}{4(x^2+x)\sqrt{x^2+x}}$

		-1		0	
f					
f'	$-$	-1	not defined	0	$+$
f''	$-$	-1		0	$-$



36.

$y = 2x - \tan x \qquad -\frac{\pi}{2} < x < \frac{\pi}{2}$

↳ as $x \to \pm\frac{\pi}{2}$

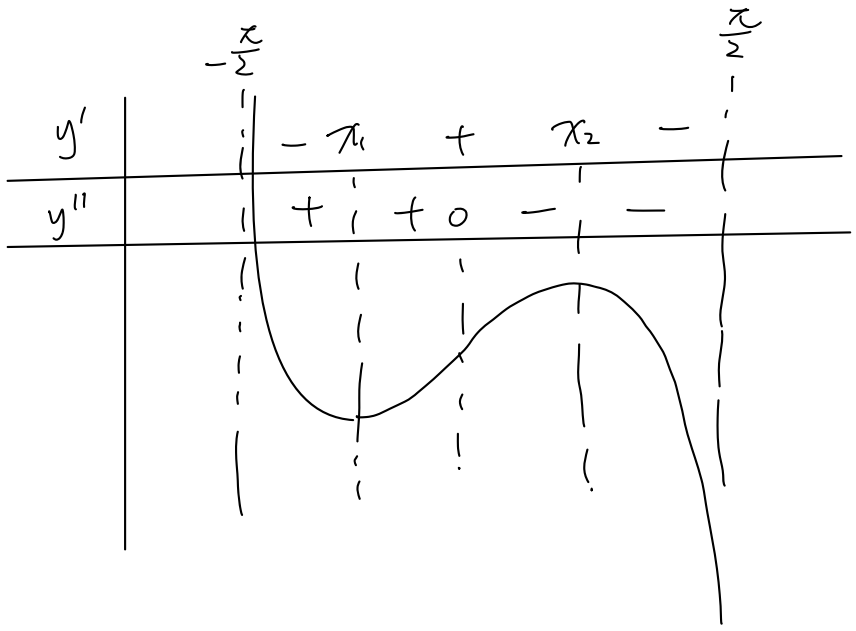
$y' = 2 - \sec^2 x$ ↳ let $y'=0$ $\tan x \to \mp\infty$

↳ $\frac{1}{\cos^2 x} = 2 \Rightarrow \cos^2 x = \frac{1}{2}$, $x = \cos^{-1}\left(\pm\frac{1}{\sqrt{2}}\right) = x_1$ & x_2

$y'' = -2\sec x \sec x \tan x = -2\sec^2 x \tan x = -2\frac{\sin x}{\cos^3 x}$

$x = 0$

有反曲点



45. $y = \frac{1}{x} + \ln x \quad \rightarrow \quad x = 0$ DNE

$$y' = -\frac{1}{x^2} + \frac{1}{x} = \frac{x-1}{x^2} = 0$$

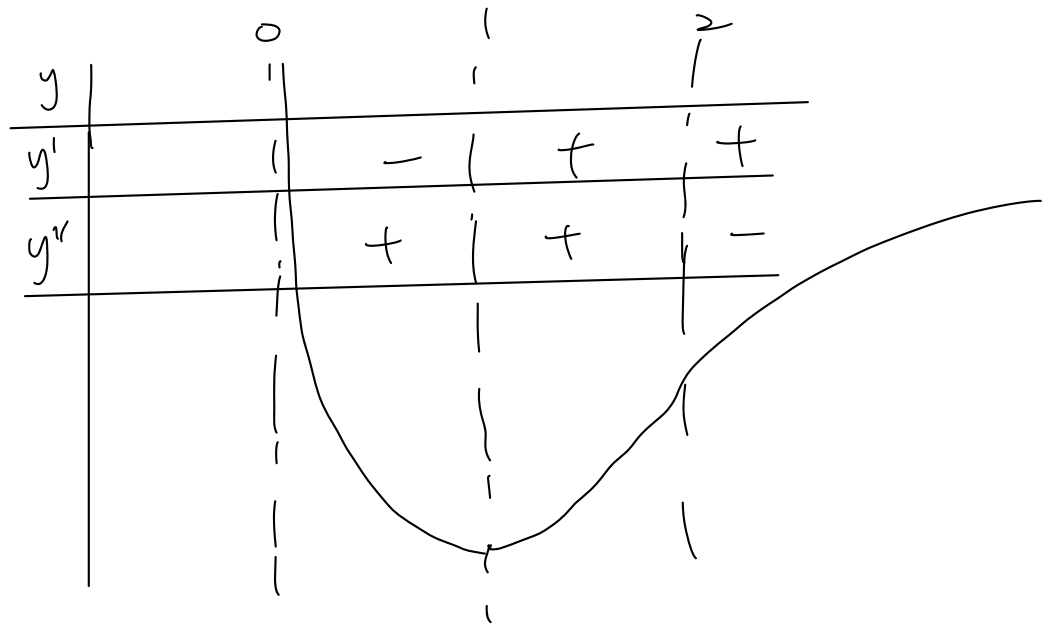
$x = 1$ 有 max or min

0 DNE

$$y'' = \frac{x^2 - (x-1)\times 2x}{x^4} = \frac{-x^2+2x}{x^4} = \frac{-x+2}{x^3}$$

$x = 2$ 有反曲点

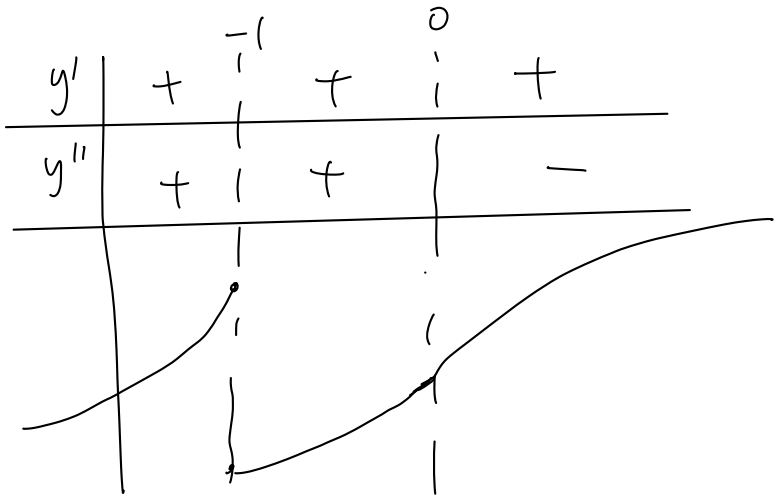
0 DNE



54. $y = \tan^{-1}\left(\frac{x-1}{x+1}\right)$ $x = -1$ DNE

$$y' = \frac{1}{1+\left(\frac{x-1}{x+1}\right)^2} = \frac{1}{x^2+1}$$

$$y'' = -\frac{2x}{(x^2+1)^2}$$ $x=0$ 有反曲点



76. $y = \sqrt{x^2+4x}$, $y' = \frac{x+2}{\sqrt{x^2+4x}}$ → 在 $(0,-4)$ DNE

$$\lim_{x\to\infty} \frac{y}{x} = \lim_{x\to\infty} \frac{\sqrt{x^2+4x}}{x} = 1$$

$$\lim_{x\to-\infty} \frac{y}{x} = \lim_{x\to-\infty} \frac{\sqrt{x^2+4x}}{x} = -1$$

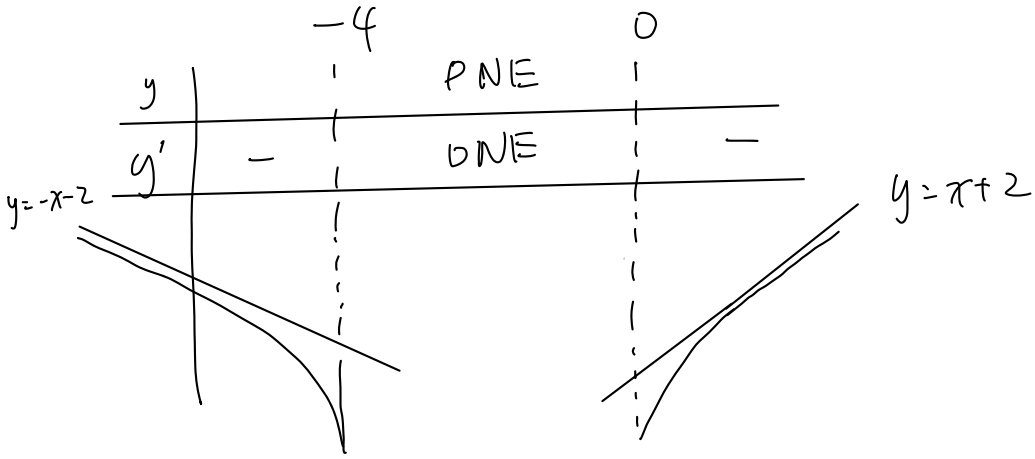
) 斜率為 ± 1

let $y_1 = x + b_1$

$y_2 = -x + b_2$

$\lim_{x\to\infty} y_1' = \lim_{x\to\infty} y' \Rightarrow b_1 = 2$

$\lim_{x\to-\infty} y_1' = \lim_{x\to-\infty} y' \Rightarrow b_2 = -2$


-4
0
y
PNE
y'
-
DNE
-
y=-x-2
y=x+2