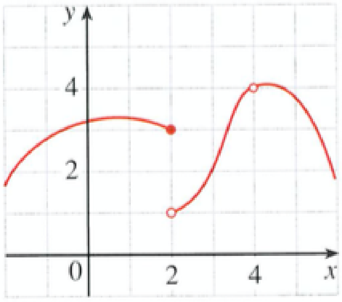
4. Use the given graph of f to state the value of each quantity, if it exists. If it does not exist, explain why.

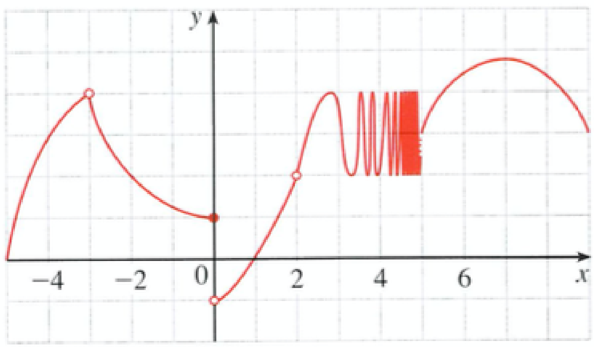
(a) $\lim_{x\to 2^-} f(x)$ (b) $\lim_{x\to 2^+} f(x)$ (c) $\lim_{x\to 2} f(x)$ (d) $f(2)$ (e) $\lim_{x\to 4} f(x)$ (f) $f(4)$



Sol.
(a) As x approaches 2 from the left, the values of $f(x)$ approach 3, so $\lim_{x\to 2^-} f(x) = 3$.
(b) As x approaches 2 from the right, the values of $f(x)$ approach 1, so $\lim_{x\to 2^+} f(x) = 1$. (Note that you don't need to consider the exact value at $x = 2$ when finding the limit.)
(c) $\lim_{x\to 2} f(x)$ does not exist since the left-hand limit does not equal the right-hand limit. (This will be the most common way to argue whether a limit exists.)
(d) From the graph, $f(2) = 3$.
(e) As x approaches 4 from the both side, the values of $f(x)$ approach 4, so $\lim_{x\to 2} f(x) = 4$.
(f) There is no value of $f(x)$ when $x = 4$, so $f(4)$ does not exist.

6. For the function h whose graph is given, state the value of each quantity, if it exists. If it does not exist, explain why.

(a) $\lim_{x\to -3^-} h(x)$ (b) $\lim_{x\to -3^-} h(x)$ (c) $\lim_{x\to -3^+} h(x)$ (d) $h(-3)$ (e) $\lim_{x\to -3} h(x)$ (f) $\lim_{x\to 0^-} h(x)$
(g) $\lim_{x\to 0^+} h(x)$ (h) $h(0)$ (i) $\lim_{x\to 2} h(x)$ (j) $h(2)$ (k) $\lim_{x\to 5^+} h(x)$ (l) $\lim_{x\to 5^-} h(x)$



Sol.
(a) $h(x)$ approaches 4 as x approaches -3 from the left, so $\lim_{x\to -3^-} h(x) = 4$.
(b) $h(x)$ approaches 4 as x approaches -3 from the right, so $\lim_{x\to -3^+} h(x) = 4$.
(c) $\lim_{x\to -3^+} h(x) = 4$ because the limits in part (a) and (b) are equal.
(d) $h(-3)$ is not defined, so it doesn't exist.
(e) $h(x)$ approaches 1 as x approaches 0 from the left, so $\lim_{x\to 0^-} h(x) = 1$.
(f) $h(x)$ approaches -1 as x approaches 0 from the right, so $\lim_{x\to 0^+} h(x) = -1$.

(g) $\lim_{x\to 0} h(x)$ does not exist because the limits in part (e) and (f) are not equal.

(h) $h(0) = 1$ since the point (0,1) is on the graph of h.

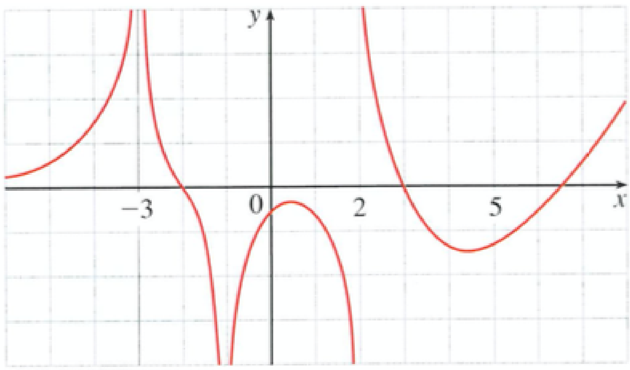
(i) Since $\lim_{x\to 2^-} h(x) = 2 = \lim_{x\to 2^+} h(x)$, we have $\lim_{x\to 2} h(x) = 2$.

(j) $h(2)$ is not defined, so it doesn't exist.

(k) $h(x)$ approaches 3 as x approaches 5 from the right, so $\lim_{x\to 5^+} h(x) = 3$.

(l) $h(x)$ doesn't approach any number as x approaches 5 from the left, so $\lim_{x\to 5^-} h(x)$ doesn't exist.

8. For the function A whose graph is shown, state the following.
(a) $\lim_{x\to -3} A(x)$ (b) $\lim_{x\to 2^-} A(x)$ (c) $\lim_{x\to 2^+} A(x)$ (d) $\lim_{x\to -1} A(x)$ (e) The equations of the vertical asymptotes.



Sol.

(a) $\lim_{x\to -3} A(x) = \infty$

(b) $\lim_{x\to 2^-} A(x) = -\infty$

(c) $\lim_{x\to 2^+} A(x) = \infty$

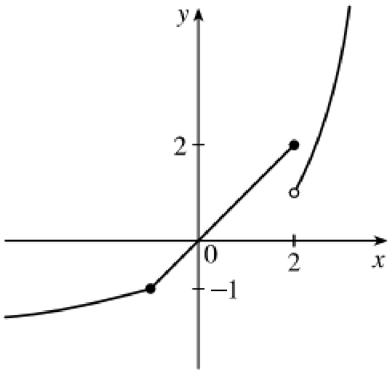
(d) $\lim_{x\to -1} A(x) = -\infty$

(e) The equations of the vertical asymptotes are $x = -3$, $x = -1$, and $x = 2$.

12. Sketch the graph of the function and use it to determine the values of a for which $\lim_{x\to a} f(x)$ exists.

$$f(x) = \begin{cases} \sqrt[3]{x} & \text{if } x \leqslant -1 \\ x & \text{if } -1 < x \leqslant 2 \\ (x-1)^2 & \text{if } x > 2 \end{cases}$$

Sol.



From the graph of f we see that $\lim_{x\to 2^-} f(x) = 2$ but $\lim_{x\to 2^+} f(x) = 1$, so $\lim_{x\to a} f(x)$ does not exist for $a = 2$. However, it exists for all other values of a. Thus, $\lim_{x\to a} f(x)$ exists for all a in $(-\infty, 2)\cup(2,\infty)$ (or $\mathbb{R}\setminus\{2\}$)

32. Determine the infinite limit.

$$\lim_{x\to 3^-} \frac{\sqrt{x}}{(x-3)^5}$$

Sol.
$\lim_{x\to 3^-} \frac{\sqrt{x}}{(x-3)^5} = -\infty$ since the numerator is positive and the denominator approaches 0 from the negative side as $x \to 3^-$.

34. Determine the infinite limit.

$$\lim_{x\to 0^+} \ln(\sin x)$$

Sol.
$\lim_{x\to 0^+} \ln(\sin x) = -\infty$ since $\sin x \to 0^+$ as $x \to 3^-$ and $\ln x \to -\infty$ as $x \to 0^+$.

36. Determine the infinite limit.

$$\lim_{x\to \pi^-} x\cot x$$

Sol.
$\lim_{x\to \pi^-} x\cot x = -\infty$ since x is positive and $\cot x \to -\infty$ as $x \to pi^-$.

38. Determine the infinite limit.

$$\lim_{x\to 3^-} \frac{x^2+4x}{x^2-x^2-3}$$

Sol.
$\lim_{x\to 3^-} \frac{x^2+4x}{x^2-x^2-3} = \lim_{x\to 3^-} \frac{x(x+4)}{(x-3)(x-1)} = -\infty$ since the numerator is positive and the denominator approaches 0 through negative values as $x \to 3^-$. (Remember to do factorization first when facing rational functions if possible!)