10. $xe^y = x - y$

$$\frac{d}{dx}(xe^y) = \frac{d}{dx}(x-y)$$

$$\Rightarrow \quad xe^y\frac{dy}{dx} + e^y = 1 - \frac{dy}{dx}$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{1-e^y}{1+xe^y}$$

12. $$\frac{d}{dx}(e^x \sin y) = \frac{d}{dx}(x+y)$$

$$\Rightarrow e^x \cos y\frac{dy}{dx} + e^x \sin y = 1 + \frac{dy}{dx}$$

$$\Rightarrow \frac{dy}{dx} = \frac{e^x \sin y - 1}{1 - e^x \cos y}$$

14. $$\frac{d}{dx}\left(\tan(x-y)\right) = \frac{d}{dx}(2xy^3+1)$$

$$\Rightarrow \sec^2(x-y)\left(1 - \frac{dy}{dx}\right) = 2y^3 + 6y^2x\frac{dy}{dx}$$

$$\sec^2(x-y) - \sec^2(x-y)\frac{dy}{dx} = 2y^3 + 6y^2x\frac{dy}{dx}$$

$$\Rightarrow \frac{dy}{dx} = \frac{\sec^2(x-y) - 2y^3}{6xy^2 + \sec^2(x-y)}$$

27. $$\frac{d}{dx}\left(ye^{\sin x}\right) = \frac{d}{dx}(x\cos y)$$

$$\Rightarrow \frac{dy}{dx}e^{\sin x} + ye^{\sin x}\cos x = \cos y - x\sin y\frac{dy}{dx}$$

$$\Rightarrow \left.\frac{dy}{dx}\right|_{\substack{x=0\\y=0}} = \left.\frac{\cos y - ye^{\sin x}\cos x}{x\sin y + e^{\sin x}}\right|_{x=0,\,y=0} = \frac{1}{1} = 1 \quad \Rightarrow L:\ y - 0 = x - 0$$

$$\Rightarrow \underline{y = x}$$

28. $\frac{d}{dx}\left(\tan(x+y)+\sec(x-y)\right)=\frac{d}{dx}(2)$

$$\Rightarrow \quad \sec^2(x+y)\times\left(1+\frac{dy}{dx}\right)+\sec(x-y)\tan(x-y)\left(1-\frac{dy}{dx}\right)=0$$

$$\left.\frac{dy}{dx}\right|_{\substack{x=\frac{\pi}{8}\\ y=\frac{\pi}{8}}}=\left.\frac{\sec^2(x+y)+\sec(x-y)\tan(x-y)}{\sec(x-y)\tan(x-y)-\sec^2(x+y)}\right|_{\substack{x=\frac{\pi}{8}\\ y=\frac{\pi}{8}}}=\frac{2+1\times 0}{1\times 0-2}=1$$

$$\Rightarrow L: \left(y-\frac{\pi}{8}\right)=\left(x-\frac{\pi}{8}\right)$$

$$\Rightarrow y=x$$

33. $x^2+y^2=(2x^2+2y^2-x)^2$

$$\frac{d}{dx}(x^2+y^2)=\frac{d}{dx}\left[(2x^2+2y^2-x)^2\right]$$

$$\Rightarrow \quad 2x+2y\frac{dy}{dx}=2(2x^2+2y^2-x)\times\left(4x+4y\frac{dy}{dx}-1\right)$$

$$\Rightarrow \left[2y-2(2x^2+2y^2-x)\times 4y\right]\frac{dy}{dx}=(4x-1)\times 2(2x^2+2y^2-x)-2x$$

$$\Rightarrow \quad \left.\frac{dy}{dx}\right|_{\substack{x=0\\ y=\frac{1}{2}}}=\left.\frac{(4x-1)\times 2(2x^2+2y^2-x)-2x}{2y-2(2x^2+2y^2-x)\times 4y}\right|_{\substack{x=0\\ y=\frac{1}{2}}}$$

$$=\frac{-1\times 2\times\frac{1}{4}}{1-2\times 2\times\frac{1}{4}\times 4\times\frac{1}{2}}=1 \quad \Rightarrow L: \left(y-\frac{1}{2}\right)=(x-0)$$

34. $x^2y^2=(y+1)^2(4-y^2)$

$$\frac{d}{dx}(x^2y^2)=\frac{d}{dx}\left[(y+1)^2(4-y^2)\right]$$

$$2xy^2+x^2 2y\frac{dy}{dx}=2(y+1)\frac{dy}{dx}(4-y^2)+(y+1)^2\times\left(-2y\frac{dy}{dx}\right)$$

$$\left.\frac{dy}{dx}\right|_{\substack{x=2\sqrt{3}\\ y=1}} = \left.\frac{2xy^2}{2(y+1)(4-y^2)-2y(y+1)^2-x^2 2y}\right|_{\substack{x=2\sqrt{3}\\ y=1}} = \frac{2\times 2\sqrt{3}}{2\times 2\times(4-1)-2\times 1(1+1)^2-12\times 2}$$

$$= -\frac{4\sqrt{3}}{20} = -\frac{\sqrt{3}}{5}$$

$$L:\quad y-1 = -\frac{\sqrt{3}}{5}\left(x-2\sqrt{3}\right)$$

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42. $\frac{d}{dx}\left(x^3-y^3\right) = \frac{d}{dx}(7)$

$\Rightarrow 3x^2-3y^2\frac{dy}{dx}=0 \quad \Rightarrow \frac{dy}{dx}=\frac{x^2}{y^2}$

$\Rightarrow \frac{d}{dx}\left(3x^2-3y^2\frac{dy}{dx}\right)=0$

$6x - 6y\frac{dy}{dx}\frac{dy}{dx} - 3y^2\frac{d^2y}{dx^2}=0$

$6x - 6y\frac{x^4}{y^4} = 3y^2\frac{d^2y}{dx^2} \quad \Rightarrow \frac{d^2y}{dx^2} = \frac{6x}{3y^2} - 6y\frac{x^4}{y^4}\times\frac{1}{3y^2}$

$= \frac{2x}{y^2} - \frac{2x^4y}{y^6} = \frac{2x}{y^2} - \frac{2x^4}{y^5}$

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50. $\sqrt{x}+\sqrt{y}=\sqrt{c}$

$\frac{d}{dx}\left(\sqrt{x}+\sqrt{y}\right) = \underbrace{\frac{d}{dx}(\sqrt{c})}_{0}$

$\frac{1}{2}x^{-\frac{1}{2}}+\frac{1}{2}y^{-\frac{1}{2}}\frac{dy}{dx}=0$

$\frac{dy}{dx} = -\frac{\sqrt{y}}{\sqrt{x}} \quad \Rightarrow L:\ (y-b) = -\left.\frac{\sqrt{y}}{\sqrt{x}}\right|_{x,y=a,b}(x-a)$

where $\sqrt{a}+\sqrt{b}=\sqrt{c}$

x-intercept : $-b = -\sqrt{\frac{b}{a}}(x-a)$

$\Rightarrow x = \left(b+\sqrt{\frac{b}{a}}a\right)\Big/\sqrt{\frac{b}{a}}$

y-intercept : $y-b = -\sqrt{\frac{b}{a}}(-a)$

$\Rightarrow y = b+\sqrt{\frac{b}{a}}a$

Sum of x & y - intercepts

$$\boxed{\frac{b+\sqrt{\frac{b}{a}}a}{\sqrt{\frac{b}{a}}} + b+\sqrt{\frac{b}{a}}a}$$

$= \underbrace{b\sqrt{\frac{a}{b}}}_{\sqrt{ab}} + a + b + \underbrace{\sqrt{\frac{b}{a}}a}_{\sqrt{ab}}$

$= a+b+2\sqrt{ab} = (\sqrt{a}+\sqrt{b})^2 = (\sqrt{c})^2$

$\boxed{=c}$

64. $x^2+4y^2=36$

$$\frac{d}{dx}\left(x^2+4y^2\right)=\frac{d}{dx}(36)$$

$$\Rightarrow \quad 2x+8y\frac{dy}{dx}=0$$

$$\Rightarrow \quad \left.\frac{dy}{dx}\right|_{\substack{x=12\\y=3}}=-\left.\frac{x}{4y}\right|_{x=12,\ y=3}=-1 \quad \Rightarrow L:\ (y-3)=(-1)(x-12)$$