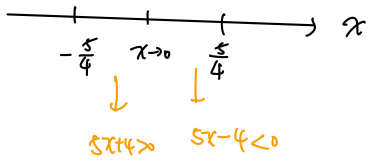
1.

(a) $$\lim_{x\to 0}\frac{|5x-4|-|5x+4|}{3x} = \lim_{x\to 0}\frac{-(5x-4)-(5x+4)}{3x} = \lim_{x\to 0}\frac{-10\cancel{x}}{3\cancel{x}} = -\frac{10}{3}$$



(b) $$\lim_{x\to 64}\frac{\sqrt{x}-8}{\sqrt[3]{x}-4} = \lim_{t\to 2}\frac{t^{6/2}-8}{t^{6/3}-4} = \lim_{t\to 2}\frac{t^3-8}{t^2-4} = \lim_{t\to 2}\frac{\cancel{(t-2)}(t^2+2t+4)}{(t+2)\cancel{(t-2)}} = \frac{4+4+4}{2+2} = 3$$

$t^6 = x$

$x\to 64 \Rightarrow t\to 2$

$$\begin{array}{r|l} & 1 \; +2 \; +4 \\ \hline 1 \; -2 & 1+0+0-8 \\ & 1-2 \\ \hline & 2+0 \\ & 2-4 \\ \hline & 4-8 \\ & 4-8 \\ \hline & 0 \end{array}$$

(c) $$\lim_{x\to 3^-}\frac{x^2-4x-12}{3x^2-10x+3} = \lim_{x\to 3^-}\frac{(x-6)(x+2)}{(3x-1)(x-3)} \longrightarrow \infty$$

$(x-3) \to 0^-$ as $x\to 3^-$

finite

and $\frac{(x-6)(x+2)}{3x-1} \to \frac{-3\times 5}{8} < 0$

(d) $$\lim_{x\to\infty}\tan^{-1}(e^{-x}\sin x)$$

$-1\leq \sin x \leq 1$ as $x\to\infty$ (finite)

$e^{-x}\to 0$ as $x\to\infty$

$\lim_{x\to\infty} e^{-x}\sin x = 0$

$$= \lim_{\phi\to 0}\tan^{-1}(\phi) = 0$$

(e) $$\lim_{x\to-\infty}\frac{\sqrt{4x^2+9}}{5x+3}$$

上下同除$-x$

$$= \lim_{x\to-\infty}\frac{\sqrt{4+\cancel{\frac{9}{x^2}}}}{-5-\cancel{\frac{3}{x}}} = \frac{\sqrt{4}}{-5} = -\frac{2}{5}$$

$\to 0$

$\to 0$

2.

(a) $x^2 - x = 4\ln x$

let $f(x) = x^2 - x - 4\ln x$

$f(2) = 4 - 2 - 4\ln 2 = -0.8$
$f(3) = 9 - 3 - 4\ln 3 = 1.6$

$f(2)f(3) < 0$, by IVT
at least one solution at $(2,3)$, s.t. $f(x) = 0$

題目說只少有2解 ⇒ 去試 有無 $f(x) > 0,\ x < 2$ or $f(x) < 0,\ x > 3$ → $f(2), f(3) > 0$ 改試

$f(\frac{1}{2}) = \frac{1}{4} - \frac{1}{2} - 4\ln\frac{1}{2}$

$= \frac{1}{4} - \frac{1}{2} + 4\ln 2 = 2.55$

這個可用 ln2, ln3 湊的

$f(\frac{1}{2})f(2) < 0$, by IVT
at least one solution at $(\frac{1}{2}, 2)$ s.t. $f(x) = 0$

(b) $f(x) = \dfrac{1+x^2}{4-x^2}$ → 有陷阱

〈錯解〉

since $f(1)\ f(3) < 0$, by IVT , ∃ at least one solution $x = c$, $1 < c < 3$, s.t. $f(c) = 0$

〈正解〉

$$f(x) = \frac{1+x^2}{4-x^2} = \frac{1+x^2}{(2+x)(2-x)}$$

⇒ 故在 (1,3) IVT不適用，無法確定有解 $x = c$ s.t. $f(c) = 0$

↓ ↓
在 $x = \pm 2$ 不連續

3. f , f' , f''

↓ f：函數
↓ f'：切線斜率
↓ f''：凹性

先猜 c 是 f ← 正確

斜率漸大又漸小 ⇒ b為 f'
凹性先正後負 ⇒ a為 f''

4. <u>Def.</u> $f'(x) = \lim\limits_{h\to 0} \dfrac{f(x+h) - f(x)}{h}$

$$= \lim_{h\to 0} \frac{\sqrt{9+(x+h)^2} - \sqrt{9+x^2}}{h} = \lim_{h\to 0} \frac{9+(x+h)^2 - (9+x^2)}{h\left(\sqrt{9+(x+h)^2} + \sqrt{9+x^2}\right)} = \lim_{h\to 0} \frac{\cancel{9}+(\cancel{x^2}+2hx+h^2) - \cancel{9} - \cancel{x^2}}{h\left(\sqrt{9+(x+h)^2} + \sqrt{9+x^2}\right)}$$

$$= \lim_{h\to 0} \frac{h^2 + 2hx}{h\left(\sqrt{9+(x+h)^2} + \sqrt{9+x^2}\right)} = \lim_{h\to 0} \frac{h + 2x}{\left(\sqrt{9+(x+h)^2} + \sqrt{9+x^2}\right)} = \frac{2x}{2\sqrt{x^2+9}} = \frac{x}{\sqrt{x^2+9}}$$