

3. 先找特殊圖形  $\Rightarrow$  (b) 有尖角, 微分後不連續  $\Rightarrow$  IV

剩 (a) (c) (d) 對 I II III

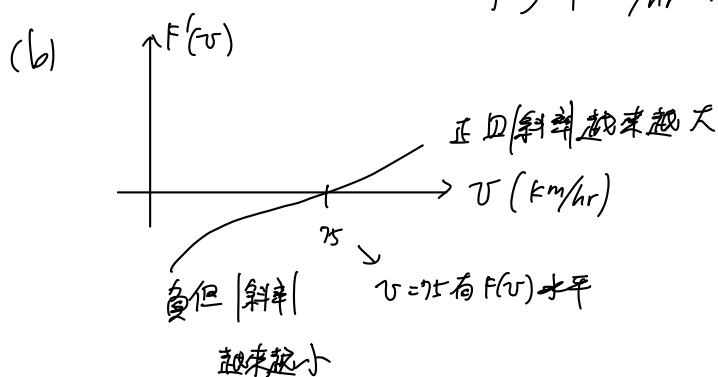
(a) 的斜率 負  $\rightarrow$  正  $\rightarrow$  負, 且在 0 處正  $\Rightarrow$  僅 II 符合

(c) 的斜率 負  $\rightarrow$  正  $\Rightarrow$  僅 I 符合

(d) 的斜率 正  $\rightarrow$  負  $\rightarrow$  正  $\rightarrow$  負  $\Rightarrow$  僅 III 符合

即為 I~IV 的函數值

14. (a)  $F'(v) = \frac{dF}{dv} \rightarrow$  需多少油耗  
 $\rightarrow$  每多 1 km/hr 的速度



(c)  $F(75)$  有  $F$  最小值,  $v=75$  km/hr 最省油

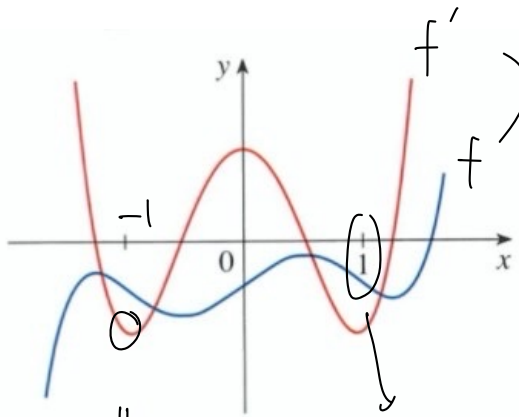
43. & 44.

$\left[ \text{可微} \Rightarrow \text{連續} \right] \Rightarrow \left[ \text{不連續} \Rightarrow \text{不可微} \right]$   
 (無唯一切線) (不連續)  
 解釋, 需為平滑曲線, 不能有尖角 or 斷角

43.  $x=1$  不連續  $\Rightarrow$  不可微

44.  $x=1$  不連續  $\Rightarrow$  不可微  
 $x=-2, 3$  尖角

47.

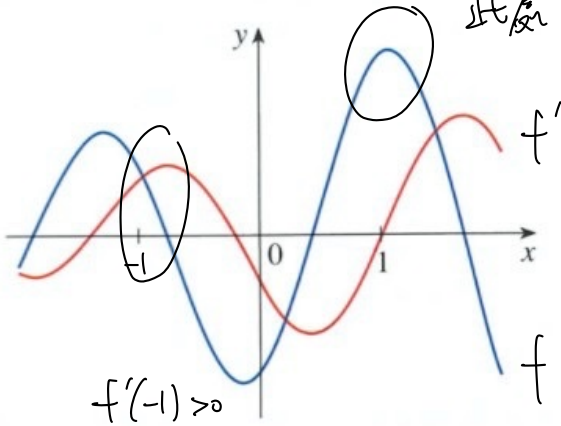


找  $f'(x) = 0$  處時  $f(x)$  有水平切線

$\downarrow$   
 $f'(-1) < 0$

此處  $f$  凹性向上,  $f''(1) > 0 \Rightarrow f''(1) > f'(-1)$

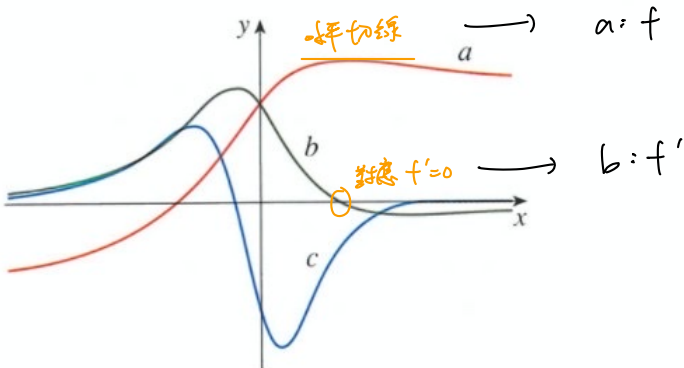
48.



此處  $f$  凹性向下,  $f''(1) < 0 \Rightarrow f'(-1) > f''(1)$

$f'(-1) > 0$

49. The figure shows the graphs of  $f$ ,  $f'$ , and  $f''$ . Identify each curve, and explain your choices.



$a = f$

$b = f'$

$\Rightarrow c = f''$

簡單檢查  $f$  左半凹向上  $f'' > 0$   
 $f$  右半凹向下  $f'' < 0$

58.

$$(a) \quad g(x) = x^{2/3}$$

$$g'(0) = \lim_{x \rightarrow 0} \frac{x^{2/3} - 0^{2/3}}{x - 0} = \lim_{x \rightarrow 0} x^{-1/3} \rightarrow \infty, \quad g'(0) \text{ DNE}$$

$$(b) \quad \forall a \neq 0, \quad g'(a) = \lim_{x \rightarrow a} \frac{x^{2/3} - a^{2/3}}{x - a} = \lim_{x \rightarrow a} \frac{(x^{2/3} - a^{2/3}) ((x^{2/3})^2 + x^{2/3} a^{2/3} + (a^{2/3})^2)}{(x-a)((x^{2/3})^2 + x^{2/3} a^{2/3} + (a^{2/3})^2)}$$

$$= \lim_{x \rightarrow a} \frac{\overset{(x+a)}{\cancel{x}} \cancel{a^2}}{\cancel{(x-a)} ((x^{2/3})^2 + x^{2/3} a^{2/3} + (a^{2/3})^2)} = \frac{2a}{3 a^{4/3}} = \frac{2}{3} a^{-1/3}$$

$$(c) \quad \text{as } a \rightarrow 0^-, \quad g'(a) = \frac{2}{3} a^{-1/3} \rightarrow -\infty$$

$$a \rightarrow 0^+, \quad g'(a) \rightarrow \infty$$

垂直切線, 但  $f'(0)$  DNE  
因左右  $g'(a)$  不同

63.

$$(a) \quad \text{even function} \quad f(x) = f(-x)$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{f(-x-h) - f(-x)}{h} = - \lim_{-h \rightarrow 0} \frac{f(-x-h) - f(-x)}{-h}$$

$$\stackrel{h=-k}{=} - \lim_{k \rightarrow 0} \frac{f(-x+k) - f(-x)}{k}$$

$$= -f'(k) \Rightarrow f' \text{ is odd function}$$

$$(b) \quad \text{odd function} \quad f(x) = -f(-x)$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{-f(-x-h) + f(-x)}{h} = \lim_{h \rightarrow 0} \frac{f(-x-h) - f(-x)}{-h}$$

$$\stackrel{h=-k}{=} \lim_{k \rightarrow 0} \frac{f(-x+k) - f(-x)}{k}$$

$$= f'(-x) \Rightarrow f' \text{ is even function}$$