2. $|f(x)-2| < 0.5$

$\Rightarrow \quad -0.5 < f(x) - 2 < 0.5$

$\Rightarrow \quad 1.5 < f(x) < 2.5 \longrightarrow$ 欲達成， x 需在 $2.6 < x < 3.8 \Rightarrow -0.4 < x-3 < 0.8$

$\Rightarrow \quad 0 < |x-3| < 0.4$

$\Rightarrow \quad \delta = 0.4$

4. $|x^2-1| < \frac{1}{2}$

$\Rightarrow \quad -\frac{1}{2} < x^2 - 1 < \frac{1}{2} \quad \Rightarrow \quad \frac{1}{2} < x^2 < \frac{3}{2} \quad \Rightarrow \quad \sqrt{\frac{1}{2}} < x < \sqrt{\frac{3}{2}}$

$\Rightarrow \quad \sqrt{\frac{1}{2}} - 1 < x - 1 < \sqrt{\frac{3}{2}} - 1$

$\Rightarrow \quad 0 < |x-1| < \sqrt{\frac{1}{2}} - 1 \quad \Rightarrow \delta = \sqrt{\frac{1}{2}} - 1$

13. (a) $|4x-8| < 0.1 \quad \Rightarrow \quad -0.1 < \underbrace{4x-8}_{4(x-2)} < 0.1 \quad \Rightarrow \quad -0.025 < x-2 < 0.025$

$\Rightarrow \quad 0 < |x-2| < 0.025$

$\Rightarrow \quad \delta = 0.025$

(b) $|4x-8| < 0.01 \quad \Rightarrow \quad -0.01 < \underbrace{4x-8}_{4(x-2)} < 0.01 \quad \Rightarrow \quad -0.0025 < x-2 < 0.0025$

$\Rightarrow \quad 0 < |x-2| < 0.0025$

$\Rightarrow \quad \delta = 0.0025$

20. $\lim_{x \to 5} \left(\frac{3}{2}x - \frac{1}{2}\right) = 7$

$\forall \varepsilon > 0, \quad \exists \delta > 0$ s.t. $\left|\frac{3}{2}x - \frac{1}{2} - 7\right| < \varepsilon$ whenever $0 < |x-5| < \delta$

$\Rightarrow \underbrace{\left|\frac{3}{2}x - \frac{15}{2}\right|}_{\frac{3}{2}|x-5|} < \varepsilon \quad \Rightarrow \quad |x-5| < \frac{2}{3}\varepsilon$

Thus take $\delta = \frac{2}{3}\varepsilon$, sufficiently small s.t. $\left|\frac{3}{2}x - \frac{1}{2} - 7\right| < \varepsilon$

26. $\lim_{x\to 0} x^3 = 0$

$\forall\, \varepsilon > 0$, $\exists\, \delta > 0$ s.t. $|x^3 - 0| < \varepsilon$ whenever $0 < |x - 0| < \delta$

$\Rightarrow |x^3| < \varepsilon \Rightarrow |x|^3 < \varepsilon \Rightarrow |x| < \varepsilon^{\frac{1}{3}}$

Thus take $\delta = \varepsilon^{\frac{1}{3}}$, sufficiently small s.t. $|x^3 - 0| < \varepsilon$

30. $\lim_{x\to 2} (x^2 + 2x - 7) = 1$

$\forall\, \varepsilon > 0$, $\exists\, \delta > 0$ s.t. $|x^2 + 2x - 7 - 1| < \varepsilon$, whenever $0 < |x - 2| < \delta$

$\Rightarrow |x^2 + 2x - 8| < \varepsilon \Rightarrow |(x-2)(x+4)| < \varepsilon \Rightarrow \underline{|x-2|\,|x+4| < \varepsilon}$

since $0 < |x-2| < \delta \Rightarrow -\delta < x - 2 < \delta \Rightarrow -\delta + 6 < x + 4 < \delta + 6$

$\Rightarrow 0 < |x+4| < 6 - \delta$

$\Rightarrow 0 < |x-2| < \dfrac{\varepsilon}{6-\delta}$

Take $\delta = \dfrac{\varepsilon}{6} < \dfrac{\varepsilon}{6-\delta}$

sufficiently small s.t. $|x^2 + 2x - 7 - 1| < \varepsilon$

41. $\dfrac{1}{(x+3)^4} > 10000$

$\forall\, \varepsilon > 0$, $\exists\, \delta > 0$ s.t. $\dfrac{1}{(x+3)^4} > 10000$, whenever $0 < |x+3| < \delta$

$\Rightarrow (x+3)^4 < \dfrac{1}{10000} \Rightarrow |(x+3)^4| < \dfrac{1}{10000}$

$\Rightarrow |x+3|^4 < \dfrac{1}{10000}$

$\Rightarrow |x+3| < \dfrac{1}{10} \Rightarrow$ Take $\delta = \dfrac{1}{10}$ sufficiently small s.t. $\dfrac{1}{(x+3)^4} > 10000$

43. $\lim_{x\to 0^+} \ln x \to -\infty$

$\forall M<0$, $\exists \delta>0$ s.t. $\ln x < M$ whenever $0<|x-0|<\delta$

$\to$ $x<e^M$ $\Rightarrow 0<|x|<e^M$ Take $\delta = e^M$ sufficiently small s.t. $\ln x < M$