

Statistics

Homework of Chapter 7.

1.Explain the following terms briefly.

(1.1) Type I and type II errors

Ans:

Type I error(型一錯誤) 虛無假設為真時，拒絕虛無假設。

Type II errors(型二錯誤) 對立假設為真時，無法拒絕虛無假設。

(1.2) Level of significance

Ans:

顯著水準 型一錯誤發生的機率

(1.3) One-sided and two-sided tests.

Ans:

One-sided tests

單尾檢定，又分為左尾檢定以及右尾檢定。

左尾檢定，希望 μ 比 μ_0 大，因此選定的顯著水準 α 全部分配於常態分配的左邊。

右尾檢定，希望 μ 比 μ_0 小，因此選定的顯著水準 α 全部分配於常態分配的右邊。

two-sided tests

雙尾檢定，不確定 μ 比 μ_0 大或是比 μ_0 小，因此選定的顯著水準 α 平均分配於常態分配的兩尾端，因此兩尾端各佔 α 的一半機率。

2. Let us consider a study of sample size 25 on systolic blood pressure. The sample mean is 120 mmHg and the population standard deviation is 20 mmHg. Test for the null hypothesis that the population mean of the systolic blood pressure is 125 mmHg, i.e., $H_0: \mu = 125$ mmHg. The alternative hypothesis is $H_1: \mu \neq 125$ mmHg.

(2.1) Perform the appropriate test for evaluating the above hypotheses at level of significance $\alpha = 0.05$ and report the corresponding p-value. What is your conclusion?

Ans:

$$H_0 = \mu = 125 \quad H_1 = \mu \neq 125$$

$$|z_0| = 1.25 \quad z_{1-\alpha/2} = 1.96 \text{(可寫可不寫 不算分)}$$

$$p\text{-value} = 2 \times 0.1056 = 0.2113$$

$$p\text{-value} > 0.05, \text{ do not reject } H_0$$

顯著水準 0.05 之下， μ_0 並非落於拒絕域，故無法拒絕虛無假設 H_0

(2.2) Compute a 95% confidence interval for the population mean of systolic blood pressure and use this interval to evaluate the above null and alternative hypotheses. What is your conclusion?

Ans:

信賴區間上界:127.8399

信賴區間下界:112.1601

$112.1601 < \mu_0 = 125 < 127.8399$, do not reject H_0

顯著水準 0.05 之下， μ_0 並非落於拒絕域，故無法拒絕虛無假設 H_0

Statistics

Homework of Chapter 8.

1. Let us consider the age (in years) of 20 respondents from a study on women in their reproductive ages. The data are listed below.

28	44	20	28	19	40	21	29	32	21
18	27	23	34	38	33	23	21	28	19

(1.1) It is of interest to know whether or not the population mean age is 28 years. What are the null hypothesis and alternative hypothesis?

Ans:

null hypothesis: $H_0 \mu=28$

alternative hypothesis: $H_1 \mu \neq 28$

(1.2) Perform the test of hypothesis using significance level $\alpha = 0.01$. What is your conclusion?

Ans:

檢定統計量 $|t_0|=0.41289$

查表 $t_{(0.05,19)} = 2.8609$

$|t_0|=0.41289 < 2.8609$

Do not reject H_0

顯著水準 0.05 之下， t_0 並非落於拒絕域，故無法拒絕虛無假設 H_0

2. In a study on cholesterol level of men and women, it is of interest to know whether or not the cholesterol level of men (m) and women (w) are the same. The sample means of men and women are 177 mg/dL and 173 mg/dL, respectively, where the sample standard deviations of men and women are 32 mg/dL and 28 mg/dL. In addition, the numbers of men and women in the study are 70 and 65, respectively. Assume that the population variances of men and women are unknown but equal, evaluate the following null and alternative hypotheses at level of significance $\alpha = 0.05$.

$$H_0 : \mu_m = \mu_w$$

$$H_1 : \mu_m \neq \mu_w$$

Does your conclusion change if you assume that variances are unknown and unequal?

Ans:

兩種情況

變異數相等

$$S_p^2 = 908.513$$

$$|t_0| = 0.7704$$

$$t_{(0.025,133)} = 1.9780$$

$$|t_0| = 0.7704 < t_{(0.025,133)} = 1.9780, \text{ Do not reject } H_0$$

顯著水準 0.05 之下， t_0 並非落於拒絕域，故無法拒絕虛無假設 H_0

變異數不相等

$$|t_0| = 0.7743$$

$$t_{(0.025,132)} = 1.9780$$

$$|t_0| = 0.7743 < t_{(0.025,132)} = 1.9780, \text{ Do not reject } H_0$$

顯著水準 0.05 之下， t_0 並非落於拒絕域，故無法拒絕虛無假設 H_0

3. The following results are obtained from two independent random samples: $\hat{p}_1 = 0.3$, $n_1 = 49$, $\hat{p}_2 = 0.5$, $n_2 = 81$. Test for the equality of two population proportions at level of significance $\alpha = 0.05$ and then summarize your conclusion.

Ans:

$$H_0: p_1 = p_2 \quad H_1: p_1 \neq p_2$$

$$\hat{p} = 0.4246$$

$$|z_0| = 2.2358$$

$$|z_0| > z_{(0.975)} = 1.96, \text{ reject } H_0$$

顯著水準 0.05 之下， z_0 落於拒絕域，故拒絕虛無假設 H_0

Statistics

Homework of Chapter 9.

1. The work by Mendel (1860) is famous for the theory and experiments on the principles of heredity. Sir R.A. Fisher (1936) reviewed Mendel's work and found a surprisingly good fit to the data. Consider two parents heterozygous for a dominant-recessive trait. Each parent has one dominant gene and one recessive gene. Mendel hypothesized that all four combinations of genes would be equally likely in the offspring. Let A denote the dominant gene and a denote the recessive gene. The two parents are Aa. The offspring should be

Genotype	Probability
AA	1/4
Aa	1/2
aa	1/4

The Aa combination has probability 1/2 since one cannot distinguish between the two cases where the dominant gene comes from one parent and the recessive gene from the other parent. In one of Mendel's experiments, he examined whether a seed was wrinkled, denoted by a, or smooth, denoted by A. By looking at offspring of these seeds, Mendel classified the seeds as aa, Aa, or AA. The results were

Genotype	Number
AA	159
Aa	321
aa	159

Do these data support the hypothesized 1:2:1 ratio? Run the appropriate test at level of significance $\alpha = 0.05$.

Ans:

$$H_0: AA:Aa:aa=1:2:1$$

$$H_a: AA:Aa:aa \neq 1:2:1$$

$$E(AA) = (159+321+159) * (1/4) = 159.75$$

$$E(Aa) = (159+321+159) * (2/4) = 319.5$$

$$E(aa) = (159+321+159) * (1/4) = 159.75$$

$$(159-159.75)^2/159=0.0035$$

$$(321-319.5)^2/321=0.0070$$

$$(159-159.75)^2/159=0.0035$$

$$\text{検定統計量: } Q=0.0035+0.0070+0.0035=0.014$$

$$Q < X^2_{(0.05,2)} = 5.9915, \text{ Do not reject } H_0$$

顯著水準 0.05 之下，Q 並非落於拒絕域，故無法拒絕虛無假設 H_0

2. A clinical trial evaluates a new compound designed to improve wound healing in trauma patients. The new compound is compared against a placebo. After treatment for 5 days with the new compound or placebo, the extent of wound healing is measured and the data are shown in the following table.

Treatment	Percent Wound Healing			
	0%-25%	26%-50%	51%-75%	76%-100%
New compound	15	37	32	41
Placebo	36	45	34	10

Is there a difference in the extent of wound healing by treatment? That is, are treatment and the percent of wound healing independent? Run the appropriate test at level of significance $\alpha = 0.05$.

Ans:

H_0 : 治療與傷口復原程度無關(獨立)

H_a : 治療與傷口復原程度有關(不獨立)

$$Q = \frac{(15 - 25.5)^2}{25.5} + \frac{(36 - 25.5)^2}{25.5} + \frac{(37 - 41)^2}{41} + \frac{(45 - 41)^2}{41} + \frac{(32 - 33)^2}{33} \\ + \frac{(34 - 33)^2}{33} + \frac{(41 - 25.5)^2}{25.5} + \frac{(10 - 25.5)^2}{25.5} = 28.3312$$

$$Q > X^2_{(0.05,3)} = 7.8147, \text{ reject } H_0$$

顯著水準 0.05 之下，Q 落於拒絕域，故拒絕虛無假設 H_0