Homework 6

Problem 1

Suppose you are a monopolist and you have two customers, Joseph and Monique. Each will buy either zero or one unit of the good you produce. Joseph is willing to pay up to $50 for your product; Monique is willing to pay up to $20. You produce this good at a constant average and marginal cost of $5.

1. If you could not engage in price discrimination, what price would you charge? How much profit would you earn?
2. If you could practice price discrimination what prices would you charge? How much profit would you earn? For simplicity, assume that if a consumer is indifferent between buying and not buying that he will buy.

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Problem 2

Suppose there are cable TV companies in your city, Astounding Cable and Broadcast Cable. They both must decide to on a high advertising budget, a moderate advertising budget, or a low advertising budget. They will make their decisions simultaneously. Their payoffs are as follows: (The unit of payoff is millions of dollars)

Astounding \ *Broadcast*	High	Medium	Low
High	(2,5)	(5,7)	(4,9)
Medium	(6,4)	(8,6)	(5,2)
Low	(1,2)	(0,5)	(3,3)

1. Does Astounding have a dominant strategy? If so, what is it?
2. Does Broadcast have a dominant strategy? If so, what is it?
3. Is there a dominant strategy equilibrium? If so, what is it?
4. Are there any Nash equilibria in this game? If so, what are they?

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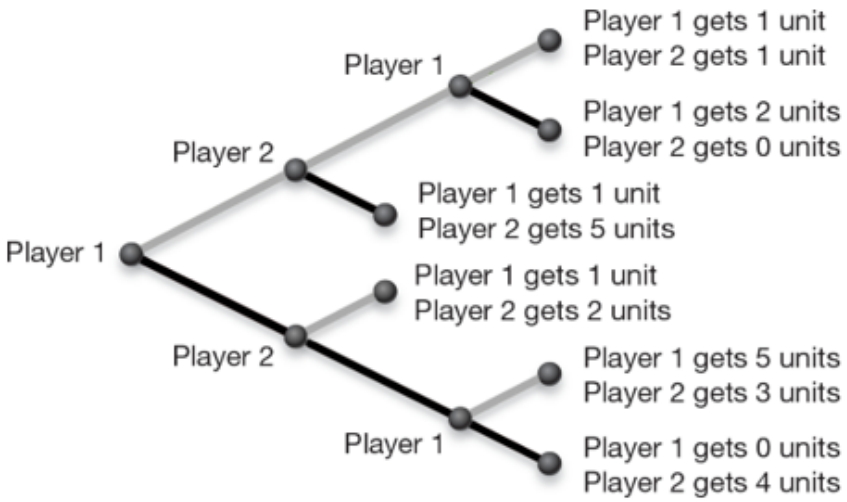
Problem 3

Consider a game with two players, 1 and 2. They play the extensive-form game summarized in the game tree below:



1. Suppose Player 1 is choosing between the gray and black for his second move. Which will he choose if:
   (a) Gray, Gray has been played.
   (b) Black, Black has been played.
2. Suppose Player 2 is choosing between gray and black, knowing the information above. Which will he choose if:
   (a) Gray has been played.
   (b) Black has been played.
3. Finally, suppose Player 1 is choosing between gray and black in the first move. Given the information above, which will he choose?
4. Now describe the path that gives an equilibrium in this extensive game.

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Problem 4

There is a market consisting of only two firms, these two competing firms must choose their quantity of production simultaneously. Each firm can either choose a High quantity of 3 or a Low quantity of 2. The demand function for this market is $P = 9 - Q$, where Q is the sum of both firms' quantities. Costs are zero, the profit is simply price times quantity. For example, if firm 1 chooses High and firm 2 chooses Low then the market price is $9 - (3 + 2) = 4$; payoff for firm 1 is 12 while payoff for firm 2 is 8.

1. Draw the complete matrix for this game.
2. Find all Nash equilibria.
3. If this game were instead played sequentially, would there be a first-mover advantage? Briefly explain.

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Problem 5

Coke and Pepsi each choose one of two prices: "Low" (P = \$2) or "High" (P = \$3). There are 50 buyers who will pick the lowest price option. However, if the prices are the same, 25 will buy from Coke and 25 from Pepsi. For simplicity, assume there are no costs, so profit is just price times quantity.

1. Draw the 2×2 payoff matrix and find all pure-strategy Nash equilibria.
2. Now assume that each company has 20 loyal buyers who buy their brand regardless of price. This leaves 10 non-loyal buyers that pick the less expensive option. Again, non-loyal buyers split evenly if the prices are the same. Draw the new payoff matrix and find all pure-strategy Nash equilibria.

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Problem 6

Suppose the world demand schedule for oil is as follows:

Price per barrel	Quantity demanded
\$50	40
\$75	30
\$125	20

There are two oil producing countries, A and B. Each will produce either 10 or 20 barrels of oil. To keep things simple, assume they can produce this oil at zero cost.

1. There are four possible outcomes: A produces 10 or 20 and B produces 10 or 20. Find each country's profit for each of these four possibilities.

2. Suppose these countries choose the quantity of oil to produce simultaneously and without consulting with one another. Show that each country will produce 20 barrels of oil and each will earn a profit of \$1,000.

3. The oil ministers realize they can do better if they collude and agree that each will produce 10. How much profit will each country earn if each produces 10 instead of 20?

4. Will Country A have an incentive to cheat and produce 20 instead of 10? Will Country B have an incentive to cheat and produce 20 instead of 10?

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Suggested Solution

Problem 1

1. The only sensible prices to consider are \$50 and \$20. If you charge more than \$50 no one will buy your product. If you charge less than \$50 but more than \$20 (e.g., \$40) Joseph will buy but Monique will not; if you are going to sell only to Joseph you should charge \$50. If you charge less than \$20 Joseph and Monique will both buy; if you are going to sell to both customers you should charge \$20. If you charge \$50 you will sell one unit of this good, your total revenues will be \$50, your total costs will be \$5, and therefore you will earn a profit of \$45. If you charge \$20 you will sell two units of this good, your total revenues will be \$40, your total costs will be \$10, and therefore you will earn a profit of \$30. Therefore the profit maximizing price is \$50.

2. If you could charge different consumers different prices (that is, you could successfully engage in price discrimination) you should charge each consumer his or her willingness to pay. This implies you should charge Joseph \$50 and Monique \$20. Your total revenues will be \$50 + \$20 = \$70, your total costs will be \$10, and you will earn a profit of \$60.

Problem 2

1. Medium is a dominant strategy for Astounding; it is a best response to any of Broadcast's strategies. For example, if Broadcast chooses High, Astounding would earn \$2 million by playing High, \$6 million by playing Medium, or \$1 million by playing Low. Therefore Astounding's best response to High is medium.

2. Broadcast does not have a dominant strategy. A strategy is a dominant strategy if it is a best response to every strategy the other players could use. For Broadcast, Low is a best response to High but Medium is a best response to Medium or Low.

3. The game does not have a dominant strategy equilibrium. In a dominant strategy equilibrium each player uses a dominant strategy. We showed in part b that Broadcast does not have a dominant strategy.

4. The game has one Nash equilibrium: Astounding and Broadcast both play Medium. A strategy combination is a Nash equilibrium if each strategy is a best response to the strategies of others. If Broadcast plays Medium then Astounding would earn \$5 million by playing High, \$8 million by playing Medium, or \$0 by playing Low; therefore Astounding's best response to Medium is Medium. A similar argument would show that Broadcast's best response to Medium is Medium.

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Problem 3

1. If both players have chosen Gray, then Player 1 would earn 1 by choosing Gray or 2 by choosing Black. Therefore, Player 1 would choose Black.
   If both players have chosen Black then Player 1 would earn 5 by choosing Gray or 0 by choosing Black. Therefore Player 1 will choose Gray.

2. If Player 1 has chosen Gray then 2 knows he will receive 0 if he chooses Gray (since, as shown above, Player 1 will choose Black on his second turn). Player 2 would earn 5 by playing Black, and so he should play Black if Player 1 has chosen Gray.
   If Player 1 has chosen Black Player 2 could earn 2 by choosing Gray. If Player 2 chooses Black, he knows that 1 will choose Gray and so 2 will earn 3. Therefore Player 2 should choose Black if Player 1 chose Black.

3. If Player 1 chooses Black, he knows that Player 2 will choose Black, that he will choose Gray on his second turn, and therefore he will earn 5. If Player 1 chooses Gray, he knows that Player 2 will choose Black, and therefore he will earn 1. Therefore Player 1 should choose Black to begin the game.

4. On the equilibrium path, Player 1 will choose Black to begin the game, Player 2 will choose Black, and Player 1 will choose Gray on his second turn.

Problem 4

1. See game matrix. If both firms choose High, then the price is \$3, and so payoff is $\$3 * 3 = \$9$ for both firms. If both firms choose Low, the price is \$5, so payoff is $\$5 * 2 = \$10$ for both firms.

$Firm1 \backslash Firm2$	High	Low
High	(9,9)	(12,8)
Low	(8,12)	(10,10)

2. (High, High) is the unique Nash equilibrium. It is also a dominant-strategy equilibrium.

3. There is no first-mover advantage because the predicted outcome is (High, High) regardless of who moves first.

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Problem 5

1. The payoffs are shown below. When both set P = \$3, each firm gets 25 buyers, and thus earn revenue is $\$3 * 25 = \$75$. If one firm undercuts the price to \$2, this firm sells to all 50 buyers, for revenue of $\$2 * 50 = \$100$. The unique pure-strategy Nash equilibrium is (Low, Low). This is also a dominant-strategy equilibrium since "Low" is a dominant strategy.

$Coke \backslash Pepsi$	High	Low
High	(\$75,\$75)	(\$0,\$100)
Low	(\$0,\$100)	(\$50,\$50)

2. When Pepsi sets a lower price, Coke still gets 20 buyers at the high price of P = \$3, for revenue of \$60; Pepsi gets all 10 non-loyal buyers, or 30 total, but at the lower price of \$2, thus they also have revenue of \$60. When the prices are the same the payoffs are the same as before as the market is split evenly. The Nash equilibrium is now (High, High). When Coke and Pepsi have loyal customers they are able to sustain an equilibrium in which they charge the higher price rather than the lower price.

$Coke \backslash Pepsi$	High	Low
High	(\$75,\$75)	(\$60,\$60)
Low	(\$60,\$60)	(\$50,\$50)

Problem 6

1. The payoff matrix below summarizes profits. Suppose both countries produce 10 and so world production is 20. We know from the demand schedule that if quantity is 20, the price of oil will be \$125 per barrel. Therefore each country will earn $10 * \$125 = \$1,250$. Similarly, if each country produces 20, world production will be 40, price will be \$50, and each country will earn $20 * \$50 = \$1,000$. Finally, if one country produces 10 and the other produces 20, then world production will be 30, price will be \$75, the country that produces 10 will earn $10 * \$75 = \$750$, and the country that produces 20 will earn $20 * \$75 = \$1,500$.

$CountryA \backslash CountryB$	10	20
10	(1250,1250)	(750,1500)
20	(1500,750)	(1000,1000)

2. You should see that producing 20 barrels is a dominant strategy for both countries. Consider Country A. If B produces 10, A earns $1,250 by producing 10 or $1,500 by producing 20. If B produces 20, A earns $750 by producing 10 or $1,000 by producing 20. In either case A maximizes profits by producing 20 barrels. A similar argument shows that 20 barrels is also a dominant strategy for B. In the dominant strategy equilibrium, each country earns $1,000.

3. If each country produces 10, each will earn $1,250.

4. Both countries will have an incentive to cheat. If B keeps its end of the agreement and produces 10, A will earn $1,250 if it also produces 10 but $1,500 if it cheats and produce 20.