Practice problems on applications.

Precalculus - Fall semester, 2021

1. We list some radioactive isotopes and their associated half-lives. Assume that each decays according to the formula $A(t) = A_0 e^{kt}$ where A_0 is the initial amount of the material and k is the decay constant. For each isotope:

   (a) Find the decay constant k.

   (b) Find a function which gives the amount of isotope A which remains after time t. (Keep the units of A and t the same as the given data.)

   (c) Determine how long it takes for 90% of the material to decay.

   A. Cobalt 60, used in food irradiation, initial amount 50 grams, half-life of 5.27 years.

   B. Phosphorus 32, used in agriculture, initial amount 2 milligrams, half-life 14 days.

   C. Chromium 51, used to track red blood cells, initial amount 75 milligrams, half-life 27.7 days.

   D. Americium 241, used in smoke detectors, initial amount 0.29 micrograms, half-life 432.7 years.

   E. Uranium 235, used for nuclear power, initial amount 1 kg grams, half-life 704 million years.

2. Under optimal conditions, the growth of a certain strain of E. Coli is modeled by the Law of Uninhibited Growth $N(t) = N_0 e^{kt}$ where N_0 is the initial number of bacteria and t is the elapsed time, measured in minutes. From numerous experiments, it has been determined that the doubling time of this organism is 20 minutes. Suppose 1000 bacteria are present initially.

   (a) Find the growth constant k.

   (b) Find a function which gives the number of bacteria $N(t)$ after t minutes.

   (c) How long until there are 9000 bacteria?

3. Yeast is often used in biological experiments. A research technician estimates that a sample of yeast suspension contains 2.5 million organisms per cubic centimeter (cc). Two hours later, she estimates the population density to be 6 million organisms per cc. Let t be the time elapsed since the first observation, measured in hours. Assume that the yeast growth follows the Law of Uninhibited Growth $N(t) = N_0 e^{kt}$.

   (a) Find the growth constant k.

   (b) Find a function which gives the number of yeast (in millions) per cc $N(t)$ after t hours.

   (c) What is the doubling time for this strain of yeast?

4. The population of Sasquatch in Bigfoot county is modeled by

   $$P(t) = \frac{120}{1 + 3.167e^{-0.05t}}$$

   where $P(t)$ is the population of Sasquatch t years after 2010.

   (a) Find and interpret $P(0)$.

   (b) Find the population of Sasquatch in Bigfoot county in 2013.

   (c) When will the population of Sasquatch in Bigfoot county reach 60? Round your answer to the nearest year.

   (d) Find and interpret the end behavior of the graph of $y = P(t)$. Check your answer using a graphing utility.

5. A pork roast was taken out of a hardwood smoker when its internal temperature had reached 180°F and it was allowed to rest in a 75°F house for 20 minutes after which its internal temperature had dropped to 170°F. Assuming that the temperature of the roast follows Newton's Law of Cooling,

   (a) Express the temperature T (in °F) as a function of time t (in minutes).

   (b) Find the time at which the roast would have dropped to 140°F had it not been carved and eaten.

6. The current i measured in amps in a certain electronic circuit with a constant impressed voltage of 120 volts is given by $i(t) = 2 - 2e^{-10t}$ where $t \geq 0$ is the number of seconds after the circuit is switched on. Determine the value of i as $t \to \infty$. (This is called the steady state current.)

7. A pot of warm soup with an internal temperature of 100°F was taken off the stove to cool in a 69°F room. After fifteen minutes, the internal temperature of the soup was 95°F.

   (a) Use Newton's Law of Cooling to write a formula that models this situation.

   (b) To the nearest minute, how long will it take the soup to cool to 80°F?

   (c) To the nearest degree, what will the temperature be after 2 and a half hours?

8. The equation $N(t) = \dfrac{500}{1 + 49e^{-0.7t}}$ models the number of people in a town who have heard a rumor after t days.

   (a) How many people started the rumor?

   (b) To the nearest whole number, how many people will have heard the rumor after 3 days?

   (c) As t increases without bound, what value does $N(t)$ approach? Interpret your answer.

9. The 1906 earthquake in San Francisco had a magnitude of 8.3 on the Richter scale. At the same time, in Japan, an earthquake with magnitude 4.9 caused only minor damage. Approximately how much more energy was released by the San Francisco earthquake than by the Japanese earthquake?

10. The concentration of hydrogen ions in a substance is denoted by $[H^+]$, measured in moles per liter. The pH of a substance is defined by the logarithmic function $\text{pH} = -\log[H^+]$. This function is used to measure the acidity of a substance. The pH of water is 7. A substance with a pH less than 7 is an acid, whereas one that has a pH of more than 7 is a base.

    Find the pH of the following substances and determine whether the substance is an acid or a base.

    A. Eggs: $[H^+] = 1.6 \times 10^{-8}$ mol/L

    B. Beer: $[H^+] = 3.16 \times 10^{-3}$ mol/L

    C. Tomato Juice: $[H^+] = 7.94 \times 10^{-5}$ mol/L

11. The demand D (in millions of barrels) for oil in an oil-rich country is given by the function $D(p) = 150 \cdot 2.7^{-0.25p}$, where p is the price (in dollars) of a barrel of oil. Find the amount of oil demanded (to the nearest million barrels) when the price is between \$15 and \$20.

12. Radiocarbon Dating Scientists can determine the age of ancient objects by the method of radiocarbon dating. The bombardment of the upper atmosphere by cosmic rays converts nitrogen to a radioactive isotope of carbon, ^{14}C, with a half-life of about 5730 years. Vegetation absorbs carbon dioxide through the atmosphere and animal life assimilates ^{14}C through food chains. When a plant or animal dies, it stops replacing its carbon and the amount of ^{14}C present begins to decrease through radioactive decay. Therefore the level of radioactivity must also decay exponentially

    (a) A discovery revealed a parchment fragment that had about 74% as much ^{14}C radioactivity as does plant material on the earth today. Estimate the age of the parchment.

    (b) Dinosaur fossils are too old to be reliably dated using carbon-14. Suppose we had a 68-million-year-old dinosaur fossil. What fraction of the living dinosaur's ^{14}C would be remaining today? Suppose the minimum detectable mass is 0.1%. What is the maximum age of a fossil that could be dated using ^{14}C?

(c) Dinosaur fossils are often dated by using an element other than carbon, such as potassium-40, that has a longer half-life (in this case, approximately 1.25 billion years). Suppose the minimum detectable mass is 0.1% and a dinosaur is dated with ^{40}K to be 68 million years old. Is such a dating possible? In other words, what is the maximum age of a fossil that could be dated using ^{40}K?

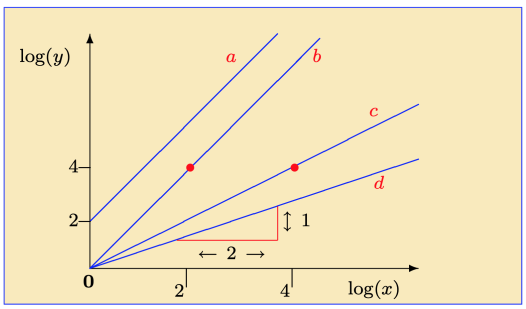
13. The rate of change of atmospheric pressure P with respect to altitude h is proportional to P, provided that the temperature is constant. At 15°C the pressure is 101.3 kPa at sea level and 87.14 kPa at $h =$ 1000 m.

    (a) What is the pressure at an altitude of 3000 m?

    (b) What is the pressure at the top of Mount McKinley, at an altitude of 6187 m?

14. In a log-log plot, the axes are $\log x$ and $\log y$.

    Which of the following lines is a log-log plot of $y = x^2$?



15. By pumping, the air pressure in a tank is reduced by 18% each second. So the percentage of air pressure remaining at time t is given by $p = 100(0.82)^t$. Plot the pressure p with respect to $0 \leq t \leq 30$ seconds in three different ways.

    (a) The rectangular coordinates [p − t].

    (b) The semi-logarithmic plot [$\log p$ − t].

    (c) The log-log plot [$\log p$ − $\log t$].