1021微甲01-04班期中考解答

1. (20%) Find the limits.

(a) $\displaystyle\lim_{x\to\infty} x^{\frac{3}{2}}\left(\sqrt{x+2}-2\sqrt{x+1}+\sqrt{x}\right)$

(b) $\displaystyle\lim_{x\to\infty}\left[x+x^2\ln\left(1-\frac{2}{x}\right)\right]$

(c) $\displaystyle\lim_{x\to 0}(\cosh 3x)^{\csc^2 x}$, where $\cosh x=\dfrac{e^x+e^{-x}}{2}$

Solution:

(a) (7%) 這一題要怎麼做呢？看到分子中間係數有個 2，不難讓人想到可以把 $\sqrt{x+1}$ 分給左右兩邊。因此

$$
\begin{aligned}
\text{原式} &= \lim_{x\to\infty} x^{\frac{3}{2}}\left[\left(\sqrt{x+2}-\sqrt{x+1}\,\right)-\left(\sqrt{x+1}-\sqrt{x}\,\right)\right]\\
&= \lim_{x\to\infty} x^{\frac{3}{2}}\left(\frac{1}{\sqrt{x+2}+\sqrt{x+1}}-\frac{1}{\sqrt{x+1}+\sqrt{x}}\right)\\
&= \lim_{x\to\infty} x^{\frac{3}{2}}\cdot\frac{\sqrt{x}-\sqrt{x+2}}{(\sqrt{x+2}+\sqrt{x+1}\,)(\sqrt{x+1}+\sqrt{x}\,)}\\
&= \lim_{x\to\infty} x^{\frac{3}{2}}\cdot\frac{-2}{(\sqrt{x+2}+\sqrt{x+1}\,)(\sqrt{x+1}+\sqrt{x}\,)(\sqrt{x}+\sqrt{x+2}\,)}\\
&= -2\lim_{x\to\infty}\frac{\sqrt{x}}{\sqrt{x+2}+\sqrt{x+1}}\cdot\lim_{x\to\infty}\frac{\sqrt{x}}{\sqrt{x+1}+\sqrt{x}}\cdot\lim_{x\to\infty}\frac{\sqrt{x}}{\sqrt{x}+\sqrt{x+2}}\\
&= -2\lim_{x\to\infty}\frac{1}{\sqrt{1+\frac{2}{x}}+\sqrt{1+\frac{1}{x}}}\cdot\lim_{x\to\infty}\frac{1}{\sqrt{1+\frac{1}{x}}+1}\cdot\lim_{x\to\infty}\frac{1}{1+\sqrt{1+\frac{2}{x}}}\\
&= -2\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\\
&= -\frac{1}{4}
\end{aligned}
$$

其中中間用了兩次的分子有理化。
另外，有許多人直接把 $x^{3/2}$ 放到分母，然後說這是不定型而使用羅畢達法則，這是不對的！理由是分子並不是 0，而是一個無法計算的量。 ■

(b) (6%) 這一題要怎麼辦呢？我們可以令 $y=1/x$，則

$$\text{原式}=\lim_{y\to 0^+}\left[\frac{1}{y}+\frac{1}{y^2}\ln(1-2y)\right]=\lim_{y\to 0^+}\left[\frac{y+\ln(1-2y)}{y^2}\right]\quad(*)$$

可以注意到這是一個不定型，故根據羅畢達法則，我們得到

$$
\begin{aligned}
(*) &\overset{L}{=} \lim_{y\to 0^+}\frac{1+\frac{-2}{1-2y}}{2y}\\
&= \lim_{y\to 0^+}\frac{1}{2y}\cdot\frac{-1-2y}{1-2y}
\end{aligned}
$$

我們發現分子是趨近於 -1 的，而分母是趨近 0^+ 的，因此 $(*)=-\infty$。
許多同學很喜歡使用 product rule，而把原題目寫成兩個 limit 的乘積，這也是不對的：

$$\text{原式}=\lim_{x\to\infty} x\left[1+x\ln\left(1-\frac{2}{x}\right)\right]=\lim_{x\to\infty} x\cdot\lim_{x\to\infty}\left[1+x\ln\left(1-\frac{2}{x}\right)\right]$$

雖然說這樣寫依然可以算出結果，不過寫法本身是有問題的。試想，$\lim_{x\to\infty} x = \infty$ 這明明不是數，為何可以寫出來運算呢？同學在使用極限的四則運算之前請三思。 ■

(c) (7%) 看到指數型的極限，通常的做法都是把底數換成 e，因此

$$\begin{aligned}
\text{原式} &= \lim_{x\to 0} \exp\left[\csc^2 x \ln(\cosh 3x)\right] \\
&= \exp\left[\lim_{x\to 0} \csc^2 x \ln(\cosh 3x)\right] \\
&= \exp\left[\lim_{x\to 0} \frac{\ln(\cosh 3x)}{\sin^2 x}\right]
\end{aligned}$$

Limit 和 e 可以交換的原因是指數函數是連續的，這麼一來只要處理其中的極限就好了！而我們發現他是一個不定型，因此可以用羅畢達法則得到

$$\begin{aligned}
\lim_{x\to 0} \frac{\ln(\cosh 3x)}{\sin^2 x} &\overset{L}{=} \lim_{x\to 0} \frac{3\sinh 3x/\cosh 3x}{2\sin x\cos x} \\
&= 3\lim_{x\to 0} \frac{\tanh 3x}{\sin 2x} \\
&\overset{L}{=} 3\lim_{x\to 0} \frac{3\operatorname{sech}^2 3x}{2\cos 2x} \\
&= 3\cdot\frac{3}{2} \\
&= \frac{9}{2}
\end{aligned}$$

因此這題的答案就是 $e^{9/2}$。
另外，如果同學對拆項法很熟的話，也可以不要馬上用羅畢達做，先拆項再羅畢達也是很漂亮的方法：

$$\begin{aligned}
\lim_{x\to 0} \frac{\ln(\cosh 3x)}{\sin^2 x} &= \lim_{x\to 0} \left[\frac{x^2}{\sin^2 x}\cdot\frac{\ln(\cosh 3x)}{\cosh 3x - 1}\cdot\frac{\cosh 3x - 1}{x^2}\right] \\
&= \lim_{x\to 0}\left(\frac{x^2}{\sin^2 x}\right)\lim_{x\to 0}\left[\frac{\ln(\cosh 3x)}{\cosh 3x - 1}\right]\lim_{x\to 0}\left(\frac{\cosh 3x - 1}{x^2}\right) \\
&= 1\cdot 1\cdot\lim_{x\to 0}\frac{\cosh 3x - 1}{x^2} \\
&\overset{L}{=} \lim_{x\to 0}\frac{3\sinh 3x}{2x} \\
&\overset{L}{=} \frac{3}{2}\lim_{x\to 0}\frac{3\cosh 3x}{1} = \frac{3}{2}\cdot 3 = \frac{9}{2}
\end{aligned}$$

其中，因為 $\cosh 3x \to 1$，而我們又知道下列極限

$$\lim_{y\to 1}\frac{\ln y}{y-1} = 1$$

我們令 $y = \cosh 3x$，就能知道我們要算的該極限值也是 1 了：

$$\lim_{x\to 0}\frac{\ln(\cosh 3x)}{\cosh 3x - 1} = 1 \quad \blacksquare$$

2. (15%) Find the derivative of the functions. (You need not simplify your answer.)

   (a) $f(x) = \log_2\left(3^x + x^4 + 5^6\right)$

   (b) $f(x) = \sin^{-1}\left(\cos^2\left(\tan x^3\right)\right)$

Solution:

(a)

$$f' = \frac{3^x \ln 3 + 4x^3}{\ln 2(3^x + x^4 + 5^6)}$$

(b)

$$f' = \frac{2\cos(\tan x^3)(-\sin(\tan x^3)) * \sec^2 x^3 * 3x^2}{\sqrt{1 - \cos^4(\tan x^3)}}$$

3. (15%) Let $f(x) = \begin{cases} |x|^x & \text{if } x \neq 0 \\ 1 & \text{if } x = 0. \end{cases}$

(a) Determine whether $f(x)$ is continuous at 0.

(b) Determine whether $f(x)$ is differentiable at 0.

Solution:

(a) To determine whether $f(x)$ is continuous at 0, that is to determine whether

$$\lim_{x\to 0} f(x) = f(0)$$

then we have to check whether

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^-} f(x) = f(0).$$

$$\Rightarrow \lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} x^x = \lim_{x\to 0^+} e^{x\ln x} = \exp \lim_{x\to 0^+} x\ln x$$

similarly,

$$\Rightarrow \lim_{x\to 0^-} f(x) = \exp \lim_{x\to 0^-} x\ln(-x)$$

Then consider

$$\lim_{x\to 0^+} x\ln x = \lim_{x\to 0^+} \frac{\ln x}{\frac{1}{x}} \to \frac{-\infty}{\infty}$$

is an indeterminate form, and then by L'Hospital rule, we can get

$$\Rightarrow \lim_{x\to 0^+} \frac{\ln x}{\frac{1}{x}} = \lim_{x\to 0^+} \frac{\frac{1}{x}}{\frac{-1}{x^2}} = \lim_{x\to 0^+} -x = 0$$

$$\Rightarrow \lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} e^0 = 1$$

similarly,

$$\Rightarrow \lim_{x\to 0^-} \frac{\ln(-x)}{\frac{1}{x}} = \lim_{x\to 0^+} \frac{\frac{1}{x}}{\frac{-1}{x^2}} = \lim_{x\to 0^-} -x = 0$$

$$\Rightarrow \lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} e^0 = 1$$

$\because \lim_{x\to 0^+} f(x) = \lim_{x\to 0^-} f(x) = 1 = f(0)$

$\therefore f(x)$ is continuous at 0.

(b) To determine whether $f(x)$ is differentiable at 0, we only need to check the limit

$$\lim_{x \to 0} \frac{f(x) - f(0)}{x - 0}$$

exists or not, or to check whether

$$\lim_{x \to 0^+} \frac{f(x) - f(0)}{x - 0} = \lim_{x \to 0^-} \frac{f(x) - f(0)}{x - 0}$$

and both limits exist.

$$\Rightarrow \lim_{x \to 0^+} \frac{f(x) - f(0)}{x - 0} = \lim_{x \to 0^+} \frac{x^x - 1}{x - 0} \to \frac{0}{0}$$

is an indeterminate form, and then by L'Hospital rule, we can get

$$\Rightarrow \lim_{x \to 0^+} \frac{x^x - 1}{x - 0} = \lim_{x \to 0^+} \frac{x^x(\ln x + 1)}{1} = -\infty$$

$$\because \frac{d}{dx} x^x = \frac{d}{dx} e^{x \ln x} = e^{x \ln x}(\ln x + x \cdot \frac{1}{x}) = x^x(\ln x + 1)$$

similarly,

$$\Rightarrow \lim_{x \to 0^-} \frac{(-x)^x - 1}{x - 0} = \lim_{x \to 0^-} \frac{(-x)^x(\ln(-x) + 1)}{1} = -\infty$$

$$\because \frac{d}{dx}(-x)^x = \frac{d}{dx} e^{x \ln(-x)} = e^{x \ln(-x)}(\ln(-x) + x \cdot \frac{-1}{-x}) = (-x)^x(\ln x + 1)$$

$\because \lim_{x \to 0^+} \frac{f(x) - f(0)}{x - 0} = -\infty$ and $\lim_{x \to 0^-} \frac{f(x) - f(0)}{x - 0} = -\infty$

$\therefore \lim_{x \to 0} \frac{f(x) - f(0)}{x - 0}$ does not exist, so $f(x)$ is not differentiable at 0.

4. (15%)
   (a) Show that the function $f(x) = x^3 + 3x + 1$ is strictly increasing on $\mathbb{R}$.
   (b) If $g(x)$ is the inverse function to the function $f(x)$ of part (a). Find $g'(5)$ and $g''(5)$.

Solution:

(a) $y' = 3x^2 + 3 > 0$, so y is strictly increasing

(b)

$$g'(5) = \frac{1}{6} \quad g''(5) = \frac{-1}{36}$$

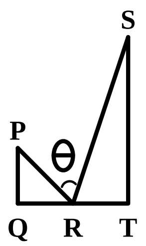
$f'(g(x))g'(x) = 1$ or $g'(f(x))f'(x) = 1$ or $3g^2(x)g'(x) + g'(x) = 1$
$f'(g(5))g'(5) = 1$ or $g'(f(1))f'(1) = 1$ or $3g^2(5)g'(5) + g'(5) = 1$; $g(5) = 1, g'(5) = 1/6$
$f''(g(x))g'^2(x) + f'(g(x))g''(x) = 0$ or $g''(f(x))f'^2(x) + g'(f(x))f''(x) = 0$ or
$6g(x)g'^2(x) + 3g^2(x)g''(x) + 3g''(x) = 1$
$f''(g(5))g'^2(5) + f'(g(5))g''(5) = 0$ or $g''(f(1))f'^2(1) + g'(f(1))f''(1) = 0$ or
$6g(5)g'^2(5) + 3g^2(5)g''(5) + 3g''(5) = 1$; $g''(5) = -1/36$

5. (10%) The minute hand on a watch is 13 mm long and the hour hand is 11 mm long. How fast is the distance between the tips of the hands changing at two o'clock?

Solution:

Let the distance between the tips be $X(t)$ and the angle be $\theta(t)$.
$X(t)^2 = 13^2 + 11^2 - 2 \cdot 13 \cdot 11 \cos\theta(t)$
$2X(t)\dfrac{dX}{dt} = -2 \cdot 13 \cdot 11(-\sin\theta(t))\dfrac{d\theta}{dt}$
$\dfrac{d\theta}{dt} = \dfrac{2\pi}{12 \cdot 60} - \dfrac{2\pi}{60}$ rad/min
By direct calculation we get
$\theta(two\ o'clock) = \dfrac{\pi}{3}$
$X(two\ o'clock) = \sqrt{147}$
$\dfrac{dX}{dt}(two\ o'clock) = \dfrac{-1573\pi}{5040}$ mm/min

6. (15%) Two vertical poles PQ and ST are secured by a rope PRS as shown in the picture.



Given that $\overline{PQ} = 1$m, $\overline{ST} = 3$m and $\overline{QT} = 2$m, we want to find the position of R such that

(a) the length of the rope PRS is maximized.

(b) the angle $\theta = \angle PRS$ is maximized.

Solution:

(a) (8%) Find the maximum, not the minimum, of the length of

$$|PR| + |RS|,$$

for the case R lie in the line QT

STEP ONE
Define the length function by

$$L(x) = \sqrt{1 + x^2} + \sqrt{3^2 + (2 - x)^2}$$

Here we setting

$$Q = (0, 0), R = (x, 0), T = (2, 0), P = (0, 1), S = (2, 3)$$

thus the domain of L(x) is

$$[0, 2]$$

STEP TWO
Take the derivative correctly, the key point is chain rule.

$$L'(x) = \frac{x}{\sqrt{1 + x^2}} + \frac{(2 - x)(-1)}{\sqrt{3^2 + (2 - x)^2}}$$

STEP THREE(no point, but relate to final answer)
Observe that you will get extreme value at crtical points of the length function

$$L(x) = \sqrt{1+x^2} + \sqrt{3^2 + (2-x)^2}$$

or the boundary of the domain of L(x)

$$[0, 2]$$

, in this case,

$$x = 0, 2$$

STEP FOUR
Find all the crtical points of the length function, since L'(x) exists on whloe domain, we just need to find what the solution of

$$L'(x) = 0$$

that is

$$\frac{x}{\sqrt{1+x^2}} + \frac{(2-x)(-1)}{\sqrt{3^2+(2-x)^2}} = \frac{x\sqrt{3^2+(2-x)^2} - (2-x)\sqrt{1+x^2}}{\sqrt{1+x^2}\sqrt{3^2+(2-x)^2}} = 0$$

A little but helpfully observation

$$\sqrt{1+x^2} > 0, \sqrt{3^2+(2-x)^2} > 0$$

then

$$(\sqrt{1+x^2})(\sqrt{3^2+(2-x)^2}) > 0$$

This make everything be simple. Since we can drop something now!!

$$\frac{x\sqrt{3^2+(2-x)^2} - (2-x)\sqrt{1+x^2}}{\sqrt{1+x^2}\sqrt{3^2+(2-x)^2}} = 0 \Leftrightarrow x\sqrt{3^2+(2-x)^2} - (2-x)\sqrt{1+x^2} = 0$$

that is the solution does not chage, but the equation mucs easy to understand !!

STEP FIVE
Slove the equation

$$x\sqrt{3^2+(2-x)^2} = (2-x)\sqrt{1+x^2}$$

correctly !! Observe that

$$x\sqrt{3^2+(2-x)^2} = (2-x)\sqrt{1+x^2} \Leftrightarrow x^2(3^2+(2-x)^2) = (2-x)^2(1+x^2)$$

For simplify, denoted that

$$Z = (2-x)^2$$

then

$$x^2(3^2+(2-x)^2) = (2-x)^2(1+x^2) \Leftrightarrow x^2(3^2+Z) = Z(1+x^2)$$

this notation make the computaion much esay

$$x^2(3^2+Z) = Z(1+x^2) \Leftrightarrow 9x^2 + Zx^2 = Z + Zx^2$$

$$\Leftrightarrow 9x^2 = Z = (2-x)^2 \Leftrightarrow 9x^2 - (4 - 4x + x^2) = 0$$

thus we only need to slove the much easy one equation

$$0 = 8x^2 + 4x - 4 = 4(2x-1)(x+1)$$

therefore we get where are all the crtical points of the length function live in, that is

$$x = \frac{1}{2}, -1$$

but -1 not lie in our domain of the length function, so just forget about it.....

Thus STEP FOUR union STEP FIVE have three point

STEP SIX

List all possible position of x that making the maximum value of the length function happen

$$L(x) = \sqrt{1 + x^2} + \sqrt{3^2 + (2 - x)^2}$$

By STEP THREE, STEP FOUR and STEP FIVE, we known

$$x = 0, 2, \frac{1}{2}$$

If we need to find the minimum value of the length, then

$$x = \frac{1}{2}$$

is the one that we want. Since by reflection on X-axis,

$$P = (0, 1) \mapsto P' = (0, -1)$$

we saw P'S is the straight line through

$$R = (\frac{1}{2}, 0)$$

thus the length

$$L(1/2) = 2\sqrt{5}$$

is the minimum value, globally!!

But we need to find the maximum value of the length function on the Domain, thus we still need to finish the comparison on

$$L(1/2) = 2\sqrt{5}, L(0) = 1 + \sqrt{13}, L(2) = 3 + \sqrt{5}$$

STEP SEVEN

Show that(or Only state it...):

$$L(1/2) = 2\sqrt{5} < L(0) = 1 + \sqrt{13} < L(2) = 3 + \sqrt{5}$$

By simple way:

$$2 = \sqrt{4} < \sqrt{5} < \sqrt{9} = 3, 5 - 4 = 1 < 4 = 9 - 5$$

conclusion :

$$\sqrt{5} \approx 2.2..... < 2.5$$

and

$$[L(1/2) = 2\sqrt{5} \approx 4.4 < 5$$
$$L(2) = 3 + \sqrt{5} \approx 5.2 > 5$$

Same method

$$3 = \sqrt{9} < \sqrt{13} < \sqrt{16} = 4, 16 - 13 = 3 < 4 = 13 - 9$$

conclusion :

$$\sqrt{13} > 3.5$$

thus

$$L(1/2) = 2\sqrt{5} \approx 4.4 < 4.5 < 1 + \sqrt{13} < 1 + 4 = 5 < 5.2 \approx 3 + \sqrt{5} = L(2)$$

Finally state : L(2) is the maximum value of the length function. Or some sentence equivalent

(b) (7%) $\theta = \pi - \arctan\frac{1}{x} - \arctan\frac{3}{2-x}$

To maximize θ, we need to minimize $g(x) = \arctan\frac{1}{x} + \arctan\frac{3}{2-x}$

$$\begin{aligned} g'(x) &= \frac{1}{1+\left(\frac{1}{x}\right)^2} \times \left(\frac{-1}{x^2}\right) + \frac{1}{1+\left(\frac{3}{2-x}\right)^2} \times \frac{3}{(2-x)^2} \\ &= \frac{-1}{x^2+1} + \frac{3}{(2-x)^2+9}. \end{aligned}$$

$g'(x) = 0 \Rightarrow x^2 + 2x - 5 = 0$ $x = \sqrt{6} - 1$

where $0 < x < \sqrt{6} - 1$, $g'(x) < 0$ and when $\sqrt{6} - 1 < x < 2$, $g'(x) > 0$

Hence $g(x)$ is minimized when $x = \sqrt{6} - 1$ which means θ is maximized at $x = \sqrt{6} - 1$

7. (20%) Let $f(x) = (x-1)^{\frac{5}{3}}(x^2-1)^{-\frac{1}{3}}$
   (a) What is the domain of $f(x)$?
   (b) Does $f(x)$ have any vertical or horizontal asymptote?
   (c) Calculate $\lim_{x\to\pm\infty}(f(x) - x)$ and find the slant asymptote of $f(x)$.
   (d) Find the intervals of increase or decrease.
   (e) Find the intervals of concavity and the inflection points.
   (f) Find the local maximum and minimum values.
   (g) Sketch the graph of $f(x)$.

Solution:

(a) domain of f is $x \in \mathbb{R}$ but $x \neq 1, -1$

(b) On the domain of f , $f(x) = \frac{(x-1)^{\frac{4}{3}}}{(x+1)^{\frac{1}{3}}}$, $\lim_{x\to\pm\infty} f(x) = \pm\infty$,so f does not have horizontal asymptote , and $\lim_{x\to-1} f(x) = \pm\infty$, so f has a vertical asymptote $x = -1$

(c)

$$\begin{aligned} &\lim_{x\to\pm\infty}(f(x) - x) \\ &= \lim_{x\to\pm\infty} \frac{(x-1)^{\frac{4}{3}} - x(x+1)^{\frac{1}{3}}}{(x+1)^{\frac{1}{3}}} \\ &= \lim_{x\to\pm\infty} \frac{(x-1)^4 - x^3(x+1)}{(x+1)^{\frac{1}{3}}[(x-1)^{\frac{8}{3}} + (x-1)^{\frac{4}{3}}x(x+1)^{\frac{1}{3}} + x^2(x+1)^{\frac{2}{3}}]} \\ &= -\frac{5}{3}, \end{aligned}$$

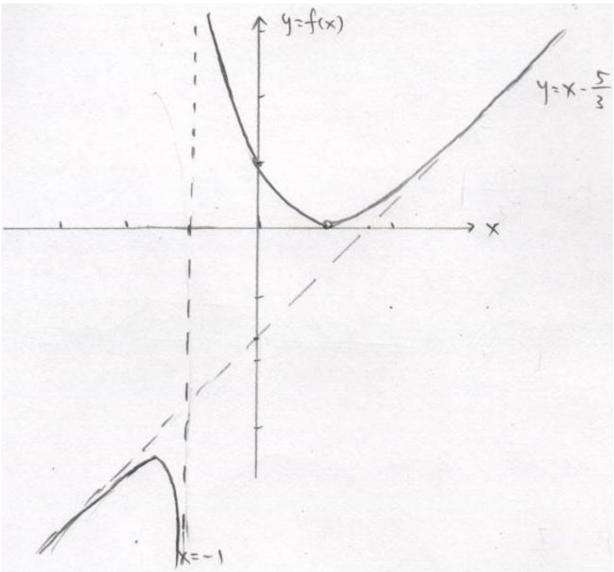
so the slant asymptote is $y = x - \dfrac{5}{3}$

(d) $f'(x) = \dfrac{\frac{4}{3}(x-1)^{\frac{1}{3}}(x+1)^{\frac{1}{3}} - \frac{1}{3}(x-1)^{\frac{4}{3}}(x+1)^{\frac{-2}{3}}}{(x+1)^{\frac{2}{3}}} = \dfrac{1}{3}(x-1)^{\frac{1}{3}}(x+1)^{\frac{-4}{3}}(3x+5)$, f is increasing on $(-\infty, -\frac{5}{3}), (1, \infty)$ and f is decreasing on $(-\frac{5}{3}, 1), (-1, 1)$

(e) $\ln f'(x) = -\ln 3 + \dfrac{1}{3}\ln|x-1| - \dfrac{4}{3}\ln|x+1| + \ln|3x+5|$, $\dfrac{f''(x)}{f'(x)} = \dfrac{1}{3} \cdot \dfrac{1}{x-1} - \dfrac{4}{3} \cdot \dfrac{1}{x+1} + \dfrac{3}{3x+5}$, then $f''(x) = \dfrac{1}{9}(x-1)^{\frac{-2}{3}}(x+1)^{-\frac{7}{3}}[(3x+5)(x+1)+9(x-1)(x+1)-4(x-1)(3x+5)] = \dfrac{16}{9}(x-1)^{\frac{-2}{3}}(x+1)^{-\frac{7}{3}}$, f concave on $(-1, 1), (1, \infty)$ and f concave down on $(-\infty, -1)$, f does not have inflection point

(f) critial point is $x = -\dfrac{5}{3}$, and $f''(-\dfrac{5}{3}) < 0$, so f has local maximum $f(-\dfrac{5}{3}) = -\dfrac{8\sqrt[3]{4}}{3}$

(g)



(a)2分 1和-1各1分 (b)2分 水平和鉛直漸近線各1分 (c)3分 算對limit有2分,寫出斜漸近線有1分 (d)3分 算對f的一次微分有2分,寫出遞增遞減區間有1分 (e)4分 算對f的二次微分有2分 寫出上凹下凹區間有1分 寫出沒有反曲點有1分

(f)2分 寫出f(x)在哪裡有局部極大值有1分,算出來有1分 未寫出f(x)沒有局部極小值不扣分 (g)4分 圖形若有畫出鉛直漸近線有1分,畫對斜漸近線有1分 X=1畫出未定義空心點有1分 剩下的1分就是圖形大致上的樣子