1042微甲06-10班期中考解答和評分標準

1. (10%) Find the sum of the series given below:

   (a) (5%) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{2^n}\cos(\frac{n\pi}{2})$; (b) (5%) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n(n+2)}$.

Solution:

(a)

$$\sum_{n=1}^{\infty} \frac{1}{2^n}\cos(\frac{n\pi}{2}) = \sum_{k=1}^{\infty} \frac{1}{4^k}(-1)^k \text{ (3 points)}$$

$$= \frac{-\frac{1}{4}}{1-(-\frac{1}{4})} = -\frac{1}{5} \text{ (2 points)}$$

(b)

$$\sum_{n=1}^{\infty} \frac{1}{n(n+2)} = \sum_{n=1}^{\infty} \frac{1}{2}(\frac{1}{n} - \frac{1}{n+2}) \text{ (2 points)}$$

$$= \frac{1}{2}\left[(\frac{1}{1}-\frac{1}{3}) + (\frac{1}{2}-\frac{1}{4}) + (\frac{1}{3}-\frac{1}{5}) + (\frac{1}{4}-\frac{1}{6}) + \ldots\right] \text{ (1 point)}$$

$$= \frac{1}{2}(1+\frac{1}{2}) = \frac{3}{4} \text{ (2 point)}$$

2. (10%) Consider the power series $\displaystyle\sum_{n=1}^{\infty} n^{\sqrt{n}}\frac{x^n}{100^n}$.

   (a) (6%) Find the radius of convergence of the given series by the Root Test.

   (b) (4%) Find the interval of convergence of the given series.

Solution:

(a)

Let $a_n = n^{\sqrt{n}}\dfrac{x^n}{100^n}$

By the Root Test

$$\lim_{n\to\infty} \sqrt[n]{|a_n|} = \lim_{n\to\infty} n^{\frac{1}{\sqrt{n}}}|\frac{x}{100}| \text{ (2 points)}$$

Note that $\displaystyle\lim_{n\to\infty} n^{\frac{1}{\sqrt{n}}} = \lim_{n\to\infty} e^{\frac{1}{\sqrt{n}}\ln n} = e^{\lim_{n\to\infty}\frac{\ln n}{\sqrt{n}}}$

$$\Rightarrow \lim_{n\to\infty} \frac{\ln n}{\sqrt{n}} = \lim_{n\to\infty} \frac{\frac{1}{n}}{\frac{1}{2}n^{-\frac{1}{2}}} = 0 \text{ (2 points)}$$

$$\Rightarrow \lim_{n\to\infty} n^{\frac{1}{\sqrt{n}}} = e^0 = 1$$

$$\lim_{n\to\infty} \sqrt[n]{a_n} = |\frac{x}{100}| < 1 \text{ (1 point)}$$

The radius of convergence is 100 (1 point)

(b)

At $x = 100$, $\displaystyle\lim_{n\to\infty} n^{\sqrt{n}} \neq 0$, Div. (1 point)

At $x = -100$, $\displaystyle\lim_{n\to\infty} n^{\sqrt{n}} \neq 0$, Div. (1 point)

The interval of convergence is $(-100, 100)$ (2 points)

3. (12%)

(a) (6%) Find the Maclaurin series for $\cos^{-1} x$.

(b) (3%) Find the radius of convergence of (a) by the Ratio Test.

(c) (3%) Find $\lim_{x\to 0^+} \dfrac{\frac{\pi}{2} - \cos^{-1} x - x}{x^3}$.

Solution:

(a)

$(\cos^{-1} x)' = \dfrac{-1}{\sqrt{1-x^2}}$ (1%)$= -\sum_{n=0}^{\infty} \begin{pmatrix} \frac{-1}{2} \\ n \end{pmatrix} (-x^2)^n$ (3%)

$\Rightarrow \cos^{-1} x = \sum_{n=0}^{\infty} \begin{pmatrix} \frac{-1}{2} \\ n \end{pmatrix} (-1)^{n+1} \dfrac{x^{2n+1}}{2n+1}$(1%) $+C$

substitue $x = 0$ into equation, we have $C = \dfrac{\pi}{2}$(1%)

$\Rightarrow \cos^{-1} x = \dfrac{\pi}{2} + \sum_{n=0}^{\infty} \begin{pmatrix} \frac{-1}{2} \\ n \end{pmatrix} (-1)^{n+1} \dfrac{x^{2n+1}}{2n+1}$

(b)

Let $\cos^{-1} x = \dfrac{\pi}{2} + \sum_{n=0}^{\infty} \begin{pmatrix} \frac{-1}{2} \\ n \end{pmatrix} (-1)^{n+1} \dfrac{x^{2n+1}}{2n+1} = \sum_{k=0}^{\infty} a_k$

then $a_k = \begin{pmatrix} \frac{-1}{2} \\ k-1 \end{pmatrix} (-1)^k \dfrac{x^{2k-1}}{2k-1}$

$\lim_{k\to\infty} |\dfrac{a_{k+1}}{a_k}|$

$= \lim_{k\to\infty} |\dfrac{\begin{pmatrix} \frac{-1}{2} \\ k \end{pmatrix} (-1)^{k+1} \frac{x^{2k+1}}{2k+1}}{\begin{pmatrix} \frac{-1}{2} \\ k-1 \end{pmatrix} (-1)^{k} \frac{x^{2k-1}}{2k-1}}|$

$= \lim_{k\to\infty} |\dfrac{(k-\frac{1}{2})(2k-1)}{k(2k+1)}| x^2$ (1%)

$= x^2$ (1%)

by ratio test

if $x^2 < 1$, series convergent

if $x^2 > 1$, series divergent

that is, series converges if $|x| < 1$ and diverges if $|x| > 1$

$\Rightarrow R = 1$ (1%)

(If your answer of (a) is wrong, you will get 0 points in (b))

(c)

$\lim_{x\to 0^+} \dfrac{\frac{\pi}{2} - \cos^{-1} x - x}{x^3}$

$= \lim_{x\to 0^+} \dfrac{\frac{\pi}{2} - (\frac{\pi}{2} - x - \frac{1}{6}x^3 - \frac{3}{40}x^5 - ...) - x}{x^3}$(1%)

$= \lim_{x\to 0^+} \dfrac{\frac{1}{6}x^3 + \frac{3}{40}x^5 + ...}{x^3}$(1%)

$= \lim_{x\to 0^+} \dfrac{1}{6} + \dfrac{3}{40}x^2 + ...$

$= \dfrac{1}{6}$(1%)

(If your answer of (a) is wrong, you will get 0 points in (c) by substitution! Another method of L'Hospital's rule is permitted.)

4. (12%) A model of the monkey of fortune and prosperity (福祿猴) is given by the following implicit function:

$$F(x,y,z) = \left(x^2 + \frac{3}{4}y^2 + z^2 - 4\right)\left(x^2 + (y+1)^2 + (z-3)^2 - 1\right) - \frac{5}{16} = 0.$$

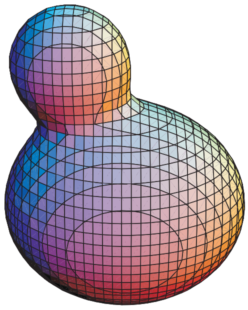

Figure 1: A model of the monkey of fortune and prosperity.

(a) (6%) Find the equation of the tangent plane to the surface at $P\left(\frac{1}{2},0,2\right)$.

(b) (6%) The surface and the plane $y=0$ intersect a curve. Find the parametric equations for the tangent line to the curve at $P\left(\frac{1}{2},0,2\right)$.

Solution:

Solution.

(a) We compute

$$\nabla F = \left(2x(x^2+(y+1)^2+(z-3)^2-1)+\left(x^2+\frac{3}{4}y^2+z^2-4\right)2x\right)\mathbf{i} + \left(\frac{3}{2}y(x^2+(y+1)^2+(z-3)^2-1)+\left(x^2+\frac{3}{4}y^2+z^2-4\right)2(y+1)\right)\mathbf{j} + \left(2z(x^2+(y+1)^2+(z-3)^2-1)+\left(x^2+\frac{3}{4}y^2+z^2-4\right)2(z-3)\right)\mathbf{k}.$$

So

$$\begin{aligned}&\nabla F\left(\frac{1}{2},0,2\right)\\ &=\left(2\cdot\frac{1}{2}\cdot\frac{5}{4}+\frac{1}{4}\cdot 2\cdot\frac{1}{2}\right)\mathbf{i}+\left(0+\frac{1}{4}\cdot 2\cdot 1\right)\mathbf{j}+\left(2\cdot 2\cdot\frac{5}{4}+\frac{1}{4}\cdot 2\cdot(-1)\right)\mathbf{k}\\ &=\frac{3}{2}\mathbf{i}+\frac{1}{2}\mathbf{j}+\frac{9}{2}\mathbf{k}\,/\!/\,3\mathbf{i}+\mathbf{j}+9\mathbf{k}.\end{aligned}$$

Hence the tangent plane equation is $3\left(x-\frac{1}{2}\right)+y+9(z-2)=0$.

知道要算∇F得 2 分，寫出對應之平面方程式得 1 分
點P帶入∇F全對得 2 分 (錯一個向量扣 1 分)，平面方程式寫對得 1 分

(b) Let $\mathbf{n}_1=3\mathbf{i}+\mathbf{j}+9\mathbf{k}$ and $\mathbf{n}_2=0\mathbf{i}+1\mathbf{j}+0\mathbf{k}$. We have $\mathbf{n}_1\times\mathbf{n_2}=-9\mathbf{i}+0\mathbf{j}+3\mathbf{k}/\!/-3\mathbf{i}+0\mathbf{j}+\mathbf{k}$. The tangent line equation is

$$\mathbf{r}(t)=\left(\frac{1}{2}-3t\right)\mathbf{i}+(2+t)\,\mathbf{k},\quad t\in\mathbb{R}.$$

寫出外積得 2 分，寫出對應之切線程式得 1 分
$\mathbf{n}_1\times\mathbf{n_2}$全對得 2 分 (錯一個向量扣 1 分)，切線方程式寫對得 1 分

Solution 2.

(a) We compute

$$\begin{aligned}\frac{\partial F}{\partial x}&=2x(x^2+(y+1)^2+(z-3)^2-1)+\left(x^2+\frac{3}{4}y^2+z^2-4\right)2x\\ \frac{\partial F}{\partial y}&=\frac{3}{2}y(x^2+(y+1)^2+(z-3)^2-1)+\left(x^2+\frac{3}{4}y^2+z^2-4\right)2(y+1)\\ \frac{\partial F}{\partial z}&=2z(x^2+(y+1)^2+(z-3)^2-1)+\left(x^2+\frac{3}{4}y^2+z^2-4\right)2(z-3).\end{aligned}$$

So

$$\frac{\partial F}{\partial x}\left(\frac{1}{2},0,2\right)=\frac{3}{2},\qquad \frac{\partial F}{\partial y}\left(\frac{1}{2},0,2\right)=\frac{1}{2},\qquad \frac{\partial F}{\partial z}\left(\frac{1}{2},0,2\right)=\frac{9}{2}.$$

At $P=(\frac{1}{2},0,2)$, the surface is locally be written as a function $z=z(x,y)$, so the surface satisfies $F(x,y,z(x,y))=0$. We compute

$$\frac{\partial F}{\partial x}+\frac{\partial F}{\partial z}\frac{\partial z}{\partial x}=0\Rightarrow\frac{\partial z}{\partial x}(p)=-\frac{\frac{\partial F}{\partial x}(p)}{\frac{\partial F}{\partial z}(p)}=-\frac{1}{3}$$
$$\frac{\partial F}{\partial y}+\frac{\partial F}{\partial z}\frac{\partial z}{\partial y}=0\Rightarrow\frac{\partial z}{\partial y}(p)=-\frac{\frac{\partial F}{\partial y}(p)}{\frac{\partial F}{\partial z}(p)}=-\frac{1}{9}.$$

Hence the tangent plane equation is

$$z-2=-\frac{1}{3}\left(x-\frac{1}{2}\right)-\frac{1}{9}y.$$

知道要找$\frac{\partial z}{\partial x}(P)$ 和 $\frac{\partial z}{\partial y}(P)$得 2 分，寫出對應之平面方程式得 1 分
$\frac{\partial z}{\partial x}(P)$ 和 $\frac{\partial z}{\partial y}(P)$算對各 1 分，平面方程式寫對得 1 分

(b) When $y=0$, the equation of the curve will be

$$F(x,z)=(x^2+z^2-4)(x^2+(z-3)^2)-\frac{5}{16}=0.$$

In this case, we have $Q(x,z)=(\frac{1}{2},2)$

$$\frac{\partial F}{\partial x}(Q)=2x(x^2+(z-3)^2)+(x^2+z^2-4)2x|_Q=\frac{3}{2}$$
$$\frac{\partial F}{\partial z}(Q)=2z(x^2+(z-3)^2)+(x^2+z^2-4)2(z-3)|_Q=\frac{9}{2}.$$

We can think the curve as $z=z(x)$ and $F(x,z(x))=0$, so we compute

$$\frac{\partial F}{\partial x}+\frac{\partial F}{\partial z}\frac{\mathrm{d}z}{\mathrm{d}x}=0\Rightarrow\frac{\mathrm{d}z}{\mathrm{d}x}(Q)=-\frac{\frac{\partial F}{\partial x}}{\frac{\partial F}{\partial z}}(Q)=-\frac{1}{3}.$$

So the tangent line equation is

$$\mathbf{r}(t)=\left(\frac{1}{2}+t\right)\mathbf{i}+\left(2-\frac{1}{3}t\right)\mathbf{k}\quad t\in\mathbb{R}.$$

知道要找隱含數之微分得 2 分，寫出對應之切線程式得 1 分
$\frac{\mathrm{d}z}{\mathrm{d}x}(Q)$算對得 2 分，切線方程式寫對得 1 分

5. (12% total, 4% each) Compute (a) the unit tangent $\mathbf{T}\left(\frac{\sqrt{2}}{2}\right)$, (b) the curvature $\kappa\left(\frac{\sqrt{2}}{2}\right)$ and (c) the arc length from $t=0$ to $t=\frac{\sqrt{2}}{2}$ of a space curve C parameterized by

$$\mathbf{r}(t)=\left(\frac{\sqrt{2}}{2}t-\frac{1}{2}t^2\right)\mathbf{i}+\left(\frac{\sqrt{2}}{2}t+\frac{1}{2}t^2\right)\mathbf{j}+\frac{1}{3}t^3\,\mathbf{k}.$$

Solution:

(a)
$\mathbf{r}'(t) = (\frac{\sqrt{2}}{2} - t, \frac{\sqrt{2}}{2} + t, t^2)$ (1pt)
$|\mathbf{r}'(t)| = \sqrt{(\frac{\sqrt{2}}{2} - t)^2 + (\frac{\sqrt{2}}{2} + t)^2 + t^4} = t^2 + 1$ (2pts)
$\mathbf{T}(t) = \frac{\mathbf{r}'(t)}{|\mathbf{r}'(t)|} = \frac{1}{t^2+1}(\frac{\sqrt{2}}{2} - t\frac{\sqrt{2}}{2} + t, t^2)$ (3pts)
$\mathbf{T}(\frac{\sqrt{2}}{2}) = \frac{2}{3}(0, \sqrt{2}, \frac{1}{2})$ (4pts)

(b)
$\mathbf{r}''(t) = (-1, 1, 2t)$
$\mathbf{r}'(t) \times \mathbf{r}''(t) = (\sqrt{2}t + t^2, -\sqrt{2}t + t^2, \sqrt{2})$ (1pt)
$|\mathbf{r}'(t) \times \mathbf{r}''(t)| = \sqrt{2}(t^2 + 1)$ (2pts)
$k(t) = \frac{|\mathbf{r}'(t) \times \mathbf{r}''(t)|}{|\mathbf{r}'(t)|^3} = \frac{\sqrt{2}}{t^2+1}$ (3pts)
$k(\frac{\sqrt{2}}{2}) = \frac{4\sqrt{2}}{9}$ (4pts)

(c)
$L = \int_0^{\frac{\sqrt{2}}{2}} |\mathbf{r}'(t)|dt$ (1pt)
$= \int_0^{\frac{\sqrt{2}}{2}} (t^2 + 1)dt$
$= \frac{t^3}{3} + t\Big|_0^{\sqrt{2}/2}$ (2pts)
$= \frac{1}{3}\frac{2\sqrt{2}}{8} + \frac{\sqrt{2}}{2} = \frac{7}{12}\sqrt{2}$ (4pts)

6. (14%) Find the points on $-x^2 + 2y^2 + 2z^2 = 1$ that are closest to the point $(0, 2, 2)$.

Solution:
Let $f(x, y, z) = x^2 + (y-2)^2 + (z-2)^2$. Employing the Lagrange multiplier, we obtain (4pts)

$$\begin{aligned} 2x &= \lambda(-2x), \\ 2(y-2) &= \lambda(4y), \\ 2(z-2) &= \lambda(4z), \\ -x^2 + 2y^2 + 2z^2 &= 1, \end{aligned}$$

where λ is the multiplier to be determined. It is easy to solve the above system to get $x = 0$ or $\lambda = -1$, and $y = z = \frac{2}{1-2\lambda}$ (2pts).

- Case (i) $x = 0$ (2pts):
  $2(\frac{2}{1-2\lambda})^2 + 2(\frac{2}{1-2\lambda})^2 = 1 \Rightarrow \frac{16}{(1-2\lambda)^2} = 1 \Rightarrow 1 - 2\lambda = \pm 4 \Rightarrow \lambda = -\frac{3}{2}, \frac{5}{2}$.
  Then $y = z = \frac{2}{1+3} = \frac{1}{2}$ or $y = z = \frac{2}{1-5} = -\frac{1}{2}$.
- Case (ii) $\lambda = -1$ (2pts):
  $y = z = \frac{2}{1+2} = \frac{2}{3} \Rightarrow x^2 = 2(\frac{2}{3})^2 + 2(\frac{2}{3})^2 - 1 = \frac{7}{9} \Rightarrow x = \pm\frac{\sqrt{7}}{3}$.

We obtain $f(0, \frac{1}{2}, \frac{1}{2}) = \frac{9}{2}$, $f(0, -\frac{1}{2}, -\frac{1}{2}) = \frac{25}{2}$ and $f(\pm\frac{\sqrt{7}}{3}, \frac{2}{3}, \frac{2}{3}) = \frac{39}{9}$ (3pts). This shows that the closest points to $(0, 2, 2)$ are $(\pm\frac{\sqrt{7}}{3}, \frac{2}{3}, \frac{2}{3})$ (1pt).

7. (16%) Let $f(x,y) = y^3 + 3x^2y - 6x^2 - 6y^2 + 1$.

(a) (8%) Find all critical points for $f(x,y)$.

(b) (8%) Determine whether they are local maximum, local minimum, or saddle points.

Solution:

(a) $f_x(x,y) = 6xy - 12x = 6x(y-2)$ (2 points)
$f_y(x,y) = 3y^2 + 3x^2 - 12y = 3(x^2 + y^2 - 4y)$ (2 points)
For $x = 0$, $y = 0$ or 4.
For $x \neq 0$, y= 2. Therefore, $x = 2$ or -2.
Consequently, (0,0), (2,2), (-2,2) and (0,4) are creitical points. (each point was worth 1 point.)

(b) $f_{xx}(x,y) = 6y - 12$
$f_{yy}(x,y) = 6y - 12$
$f_{xy}(x,y) = 6x$
$D = f_{xx}f_{yy} - (f_{xy})^2$ (Above description was worth for total 4 points)

For (2,2) and (-2,2),
$f_{xx}f_{yy} - (f_{xy})^2 = -144 < 0$
(2,2) and (-2,2) are saddle points. (2 points)

For (0,0),
$f_{xx}f_{yy} - (f_{xy})^2 = 144 > 0$ and $f_{xx}(0,0) < 0$
(0,0) is local maximum. (1 point)

For (0,4),
$f_{xx}f_{yy} - (f_{xy})^2 = 144 > 0$ and $f_{xx}(0,4) > 0$
(0,4) is local minimum. (1 point)

8. (14%) Consider a function $z = z(x,y)$ satisfying the partial differential equation:

$$6\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial x \partial y} - \frac{\partial^2 z}{\partial y^2} = 0.$$

Suppose that $u = x - 2y$ and $v = x + ay$ will change the partial differential equation into $\dfrac{\partial^2 z}{\partial u \partial v} = 0$. Find the constant a.

Solution:

(方法一) 首先由條件可知

$$\frac{\partial u}{\partial x} = 1 \quad ; \quad \frac{\partial u}{\partial y} = -2 \quad ; \quad \frac{\partial v}{\partial x} = 1 \quad ; \quad \frac{\partial v}{\partial y} = a$$

根據多變數微分的連鎖律有以下式子

$$\frac{\partial z}{\partial x} = \frac{\partial z}{\partial u}\frac{\partial u}{\partial x} + \frac{\partial z}{\partial v}\frac{\partial v}{\partial x} = z_u + z_v$$
$$\frac{\partial z}{\partial y} = \frac{\partial z}{\partial u}\frac{\partial u}{\partial y} + \frac{\partial z}{\partial v}\frac{\partial v}{\partial y} = -2z_u + az_v$$

進一步可以

$$\frac{\partial^2 z}{\partial x^2} = \frac{\partial z_u}{\partial x} + \frac{\partial z_v}{\partial x} = \left[\frac{\partial z_u}{\partial u}\frac{\partial u}{\partial x} + \frac{\partial z_u}{\partial v}\frac{\partial v}{\partial x}\right] + \left[\frac{\partial z_v}{\partial u}\frac{\partial u}{\partial x} + \frac{\partial z_v}{\partial v}\frac{\partial v}{\partial x}\right] = z_{uu} + 2z_{uv} + z_{vv}$$

$$\begin{aligned}\frac{\partial^2 z}{\partial x \partial y} &= -2\frac{\partial z_u}{\partial x} + a\frac{\partial z_v}{\partial x} = -2\left[\frac{\partial z_u}{\partial u}\frac{\partial u}{\partial x} + \frac{\partial z_u}{\partial v}\frac{\partial v}{\partial x}\right] + a\left[\frac{\partial z_v}{\partial u}\frac{\partial u}{\partial x} + \frac{\partial z_v}{\partial v}\frac{\partial v}{\partial x}\right] \\ &= -2z_{uu} + (a-2)\, z_{uv} + au_{vv}\end{aligned}$$

$$\begin{aligned}\frac{\partial^2 z}{\partial y^2} &= -2\frac{\partial z_u}{\partial y} + a\frac{\partial z_v}{\partial y} = -2\left[\frac{\partial z_u}{\partial u}\frac{\partial u}{\partial y} + \frac{\partial z_u}{\partial v}\frac{\partial v}{\partial y}\right] + a\left[\frac{\partial z_v}{\partial u}\frac{\partial u}{\partial y} + \frac{\partial z_v}{\partial v}\frac{\partial v}{\partial y}\right] \\ &= 4z_{uu} - 4az_{uv} + a^2 z_{vv}\end{aligned}$$

因此結合這些代入原方程中可得

$$6\left(z_{uu}+2z_{uv}+z_{vv}\right)+\left(-2z_{uu}+(a-2)\,z_{uv}+az_{vv}\right)-\left(4-4az_{uv}+a^2z_{vv}\right)=0$$

化簡即

$$(5a+10)\,z_{uv}+\left(6+a-a^2\right)z_{vv}=0$$

這表明要滿足如下的條件才能確保原方程可推得$z_{uv}=0$:

$$5a+10\neq 0 \quad \text{且} \quad 6+a-a^2=0$$

從而$a=3$。(方法二) 由條件可求得

$$x=\frac{au+2v}{a+2} \quad ; \quad y=\frac{-u+v}{a+2}$$

由此按以下步驟計算偏導函數

$$\frac{\partial z}{\partial v}=\frac{\partial z}{\partial x}\frac{\partial x}{\partial v}+\frac{\partial z}{\partial y}\frac{\partial y}{\partial v}=\frac{2}{a+2}z_x+\frac{1}{a+2}z_y=\frac{1}{a+2}\left(2z_x+z_y\right)$$

因此

$$\begin{aligned}\frac{\partial^2 z}{\partial u\partial v}&=\frac{\partial}{\partial u}\left(\frac{\partial z}{\partial v}\right)=\frac{1}{a+2}\left(2\frac{\partial z_x}{\partial u}+\frac{\partial z_y}{\partial u}\right)\\&=\frac{1}{a+2}\left[2\left(\frac{\partial z_x}{\partial x}\frac{\partial x}{\partial u}+\frac{\partial z_x}{\partial y}\frac{\partial y}{\partial u}\right)+\left(\frac{\partial z_y}{\partial x}\frac{\partial x}{\partial u}+\frac{\partial z_y}{\partial y}\frac{\partial y}{\partial u}\right)\right]\\&=\frac{1}{(a+2)^2}\left(2az_{xx}+(a-2)\,z_{xy}-z_{yy}\right)=0\end{aligned}$$

從而比較係數可得

$$2a=6 \quad ; \quad (a-2)=1$$

如此算得 $a=3$。

評分標準:

(1) 完整回答者得14分。

(2) 二階偏微分的連鎖律計算過程給部分分,佔8分。

(3) 根據求得a的部分進行處理,佔3分。

(4) 一階偏微分的計算佔3分。