Chapter 4 :The flow physics from small to large Reynolds number

Method of solutions :

(1) Exact solutions (very few)
– as the Navier-Stokes equation is nonlinear.

(2) Numerical solutions
– CFD – computational fluid dynamics.

(3) Approximate solutions (analytical methods)
– Here in this course we study the problems under condition of either very small or very large Reynolds number.

Governing equations for steady incompressible flow with constant viscosity

$$\nabla \cdot \mathbf{u} = 0$$

Could we reduce these further ?

$$\rho \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla P + \mu \nabla^2 \mathbf{u}$$

subject to no-slip boundary conditions at solid wall.

Scaling analysis :

velocity scale : U_s length scale : L_s pressure scale : P_s

(depends on Reynolds number)

Low Reynolds number flows (1)
(small inertia)

For low Reynolds number, we require

$$-\nabla P \sim \mu \nabla^2 \mathbf{u} \quad \Rightarrow \quad \frac{P_s}{L_s} \sim \mu \frac{U_s}{L_s^2} \quad \Rightarrow \quad P_s \sim \frac{\mu U_s}{L_s}$$

On setting

$$\hat{\mathbf{u}} = \frac{\mathbf{u}}{U_s}, \quad \hat{\mathbf{x}} = \frac{\mathbf{x}}{L_s}, \quad \hat{\nabla} = L_s \nabla, \quad \hat{P} = \frac{P}{\mu U_s / L_s}$$

The governing equations:

$$\hat{\nabla} \cdot \hat{\mathbf{u}} = 0$$

$$\mathbf{R}\hat{\mathbf{u}} \cdot \hat{\nabla} \hat{\mathbf{u}} = -\hat{\nabla} \hat{P} + \hat{\nabla}^2 \hat{\mathbf{u}}$$

with Reynolds number:

$$\mathbf{R} = \frac{\rho U_s L_s}{\mu}$$

Low Reynolds number flows (2)

Physical meaning of Reynolds number :

$$\frac{\rho \mathbf{u}\cdot\nabla\mathbf{u}}{\mu\nabla^2\mathbf{u}} \sim \frac{\rho U_s \dfrac{U_s}{L_s}}{\mu \dfrac{U_s}{L_s^{\,2}}} \sim \frac{\rho U_s L_s}{\mu} = \mathrm{R}$$

As R → 0, the inertia force is negligible in comparing with the viscous force, the governing equations become

$$\hat{\nabla}\cdot\hat{\mathbf{u}} = 0$$
$$\mathbf{0} = -\hat{\nabla}\hat{P} + \hat{\nabla}^2\hat{\mathbf{u}}$$

(dimensionless form)

or

$$\nabla\cdot\mathbf{u} = 0$$
$$\mathbf{0} = -\nabla P + \mu\nabla^2\mathbf{u}$$

(dimensional form)

(called Stokes equation)

Low Reynolds number flows (3)

- Low Reynolds number flow = Stokes flow = creeping flow
- There are three situations for the Reynolds number to be small: when the characteristic velocity is small, when the characteristic length is small, and when the viscosity is large.
- Recall that the inertia force is identically zero for flows with parallel streamlines as discussed in chapter 3, such parallel flows together with the low Reynolds number flows are sometimes called the inertial-free flows (because the inertia term is neglected) or inertialless flows (in Liggett's book).
- The governing equations become **linear** when the inertia term is neglected, which implies that we can carry out further theoretical analysis under $R \ll 1$ (see Chapter 5).

國立台灣大學 應用力學研究所
Institute of Applied Mechanics
National Taiwan University

High Reynolds number flows (1)
(large inertia)

For high Reynolds number flows,

$$\rho \mathbf{u} \cdot \nabla \mathbf{u} \sim -\nabla P \quad \Rightarrow \quad \rho U_s \frac{U_s}{L_s} \sim \frac{P_s}{L_s} \quad \Rightarrow \quad P_s \sim \rho U_s^{\,2}$$

On setting: $\hat{\mathbf{u}} = \dfrac{\mathbf{u}}{U_S}$, $\hat{\mathbf{x}} = \dfrac{\mathbf{x}}{L_s}$, $\hat{\nabla} = L_s \nabla$, $\hat{P} = \dfrac{P}{\rho U_s^{\,2}}$,

The governing equations are: $\hat{\nabla} \cdot \hat{\mathbf{u}} = 0$

$$\hat{\mathbf{u}} \cdot \hat{\nabla} \hat{\mathbf{u}} = -\hat{\nabla} \hat{P} + \frac{1}{\mathbf{R}} \hat{\nabla}^2 \hat{\mathbf{u}}$$ (2nd order eqn.)

As $R \to \infty$,

$$\hat{\nabla} \cdot \hat{\mathbf{u}} = 0$$

$$\hat{\mathbf{u}} \cdot \hat{\nabla} \hat{\mathbf{u}} = -\frac{1}{\rho} \hat{\nabla} \hat{P}$$

(1st order eqn.)

or

$$\nabla \cdot \mathbf{u} = 0$$

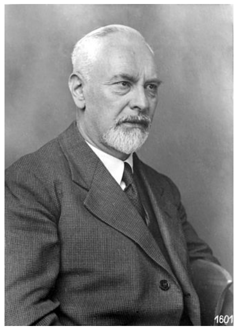
$$\rho \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla P$$

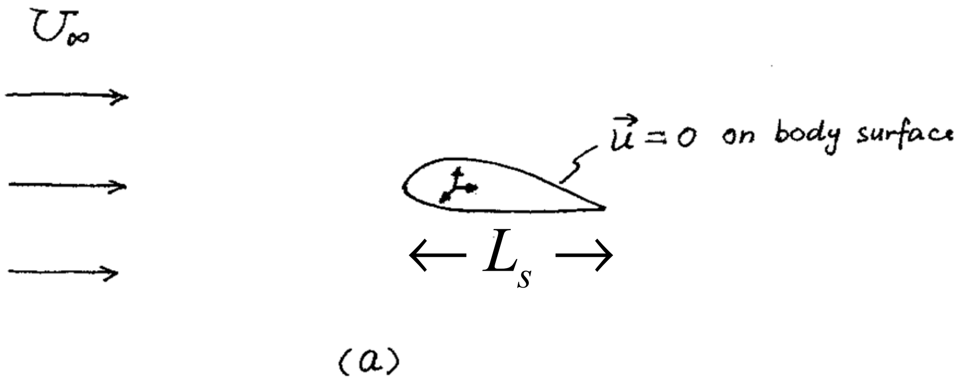
(Euler equation)

(but cannot satisfy the no-slip condition ?)

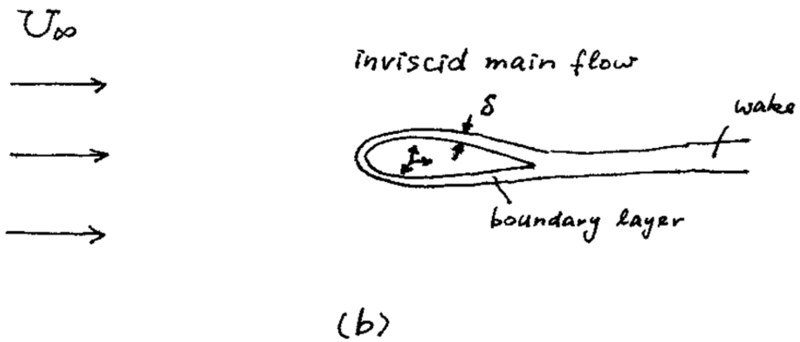
High Reynolds number flows (2)

- In order to satisfy the no-slip condition (the real situation) for flow with large Reynolds number, we must retain at least some part of the viscous term in the equation of motion.
- Proposed that there exists a thin region next to the solid boundary with thickness δ, where the viscous force is not negligible and of the same order as the inertia force. Such thin region is called the boundary layer, and was first proposed by Ludwig Prandtl at 1904.





$\delta << L_s$



Summary :

Steady incompressible flow with constant viscosity

$\nabla \cdot \mathbf{u} = 0$ (Continuity equation)

$\rho \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla P + \mu \nabla^2 \mathbf{u}$ (Navier-Stokes equation) (2^{nd}-order)

10^{-5} 10^{-4} 10^{-3} 10^{-2} 10^{-1} 1 10^{1} 10^{2} 10^{3} 10^{4} 10^{5} $\mathbf{R} = \dfrac{\rho U_s L_s}{\mu}$ **(Reynolds Number)**

Low Reynolds number flows ($\mathbf{R} \ll 1$) :

$\nabla \cdot \mathbf{u} = 0$

$0 \approx -\nabla p + \mu \nabla^2 \mathbf{u}$ (Stokes equation)

(2^{nd}-order)

High Reynolds number flows ($\mathbf{R} \gg 1$) : **(Two regions)**

$\nabla \cdot \mathbf{u} = 0$

$\rho \mathbf{u} \cdot \nabla \mathbf{u} \approx -\nabla p$ (Euler equation) $\rightarrow$ **Inviscid main flow**

(1^{st} - order ...... cannot satisfy the no - slip BC)

$\nabla \cdot \mathbf{u} = 0$

$\rho \mathbf{u} \cdot \nabla \mathbf{u} \approx -\nabla p + \mu \dfrac{\partial^2 \mathbf{u}}{\partial y^2}$ (parabolized Navier-Stokes equation)

(2^{nd}-order equation) $\rightarrow$ **Boundary layer**

(y is in the direction across the boundary layer)

High Reynolds number flow (3)

Different length scales in different directions :

x (streamwise) direction $-\ L_x$

y (cross streamwise) direction $-\ \delta$

z (spanwise) direction $-\ L_z$

We require : $\delta << L_x \sim L_z \sim L_s$ inside the boundary layer. **(see later)**

The viscous term in the streamwise momentum equation :

$$\mu \nabla^2 u = \mu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) \sim \mu \left(\frac{U_s}{L_s^2} + \frac{U_s}{\delta^2} + \frac{U_s}{L_s^2} \right) \sim \mu \frac{U_s}{\delta^2}$$

↑

dominated term, should be retained under R >> 1

High Reynolds number flow (4)

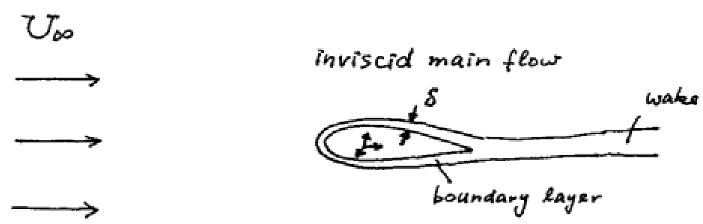
By balancing such viscous term with the inertia term,

$$\rho U_s \frac{U_s}{L_s} \sim \mu \frac{U_s}{\delta^2} \;\Rightarrow\; \frac{\delta}{L_s} \sim \sqrt{\frac{\mu}{\rho U_s L_s}} = \frac{1}{\sqrt{\mathrm{R}}} \;\Rightarrow\; \delta << L_s \;\textbf{as}\; \mathbf{R} \to \infty$$

- Thus there are **two regions** **(one with length δ and one with L_s)** having different characteristics for the flow field at large **R**.
- Most of the region is essentially inviscid, and is called the **inviscid main flow** (theme of Chapter 6).
- The viscous effect is confined in a thin region next to the wall, called the **boundary layer** (theme of chapter 7).
- The thickness of the boundary layer (length scale in "*y*-direction") is much less than its stream-wise extent (length scale along the "*x*-direction", which is of the same order as the length scale of the inviscid main flow. $(\delta << L_s)$

Discussion - High Reynolds number flow (1)

- **Mathematic**ally, the boundary layer is inserted for satisfying the no-slip boundary condition.
- **Vorticity** is generated at the wall associated with the no-slip condition, and swept downstream by the flow. The diffusion of the vorticity by the viscous effect across the stream is relatively small in comparing with the streamwise convective effect due to the imposed flow at large Reynolds number, and thus the boundary layer is kept thin although its thickness is evolving slowly downstream.
- **Wake** is also a kind of boundary layer. It contains mainly the relatively high vorticity fluid generated inside the boundary layers upstream, and is fundamentally different from that in the inviscid main flow (see Chapter 7).



Discussion - High Reynolds number flow (2)

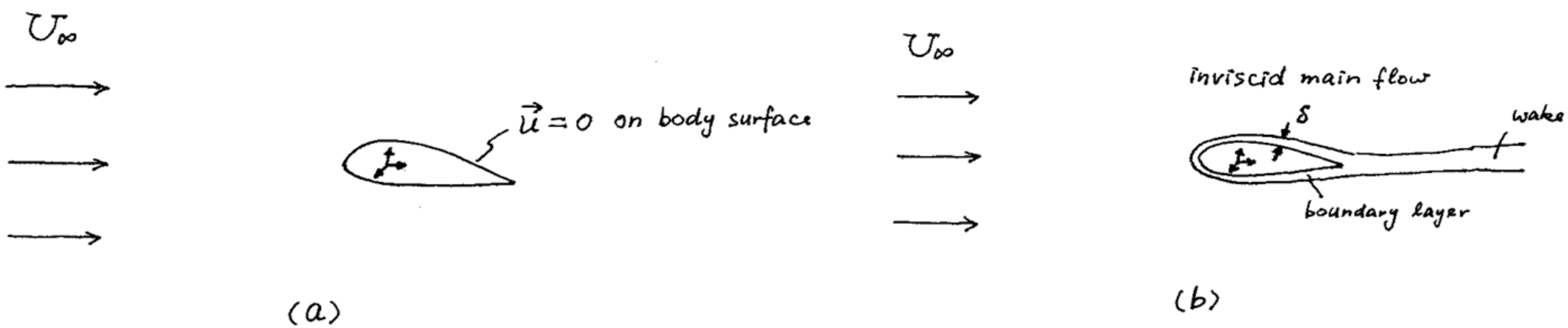
- The boundary layer region contains
  (1) the **boundary layer** next to the solid wall
  (2) the wake (and other **free shear flows** such as jet and mixing layer)
  (Boundary layer flow without solid boundary!)
- The boundary layer region has two main characteristics:
  (1) it is a **slender** region, and thus the spatial variation of the flow properties across the flow is much greater than that along the flow, and
  (2) **viscous** effect is important in this region.
- When the Reynolds number is large enough, the flows inside the boundary layer region are **turbulent**, and the thickness of the boundary layer region becomes larger in comparing with that in laminar case. However, the main characteristics remain unchanged.

Discussion - High Reynolds number flow (3)

- If the flow inside the boundary layer experiences a sufficiently severe adverse pressure gradient, it may separate from the body surface, and the boundary layer grows rapidly. The boundary layer assumption $\delta << L_x \sim L_z \sim L_s$ is not valid locally where the **boundary layer separates**. Also the boundary layer assumption fails in the wake region near the body (called the near wake). Numerical solution of the continuity and Navier-Stokes equations is required for understanding the local detailed features of such cases.
- However, the structure of the inviscid main flow can still be analyzed using the continuity and Euler equations, but the shape of the body for inviscid calculation should be modified (in principle).

Discussion - High Reynolds number flow (4)

- Therefore, in case with high Reynolds number, we may replace "the original problem governed by the continuity and Navier -Stokes equations subject to the no-slip boundary condition at solid surface as in Figure 4-1(a)" by "the problem with two regions governed separately by simpler equations as in Figure 4-1(b)". The whole flow domain is separated into two parts in Figure 4-1(b), **the inviscid main flow** and **the thin boundary layer** region. The latter includes the boundary layer next to the solid surface and the wake region in the downstream of the body.



Discussion - High Reynolds number flow (5)

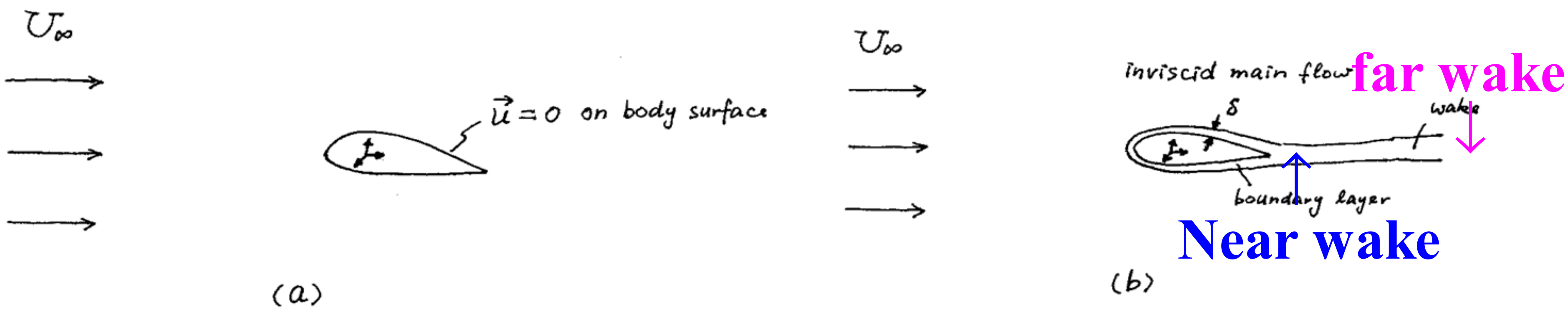
- The inviscid main flow includes most of the flow region of the original problem, and is governed by the continuity and the Euler equations (with slip boundary condition).
- The flow in the boundary layer region is governed by the so-called boundary layer equations (will be derived later), which include the continuity and a "simplified" (parabolized) form of the Navier-Stokes equation.
- For the boundary layer next to the solid surface, the boundary layer equations are solved subject to the **no-slip condition** at the solid boundary and the **matching condition** at the outer edge of the boundary layer.

Discussion - High Reynolds number flow (6)

- The matching condition enforces that the flow fields obtained in both the inviscid main flow region and the boundary layer region obtained from different sets of governing equations to be matched smoothly in a common region.

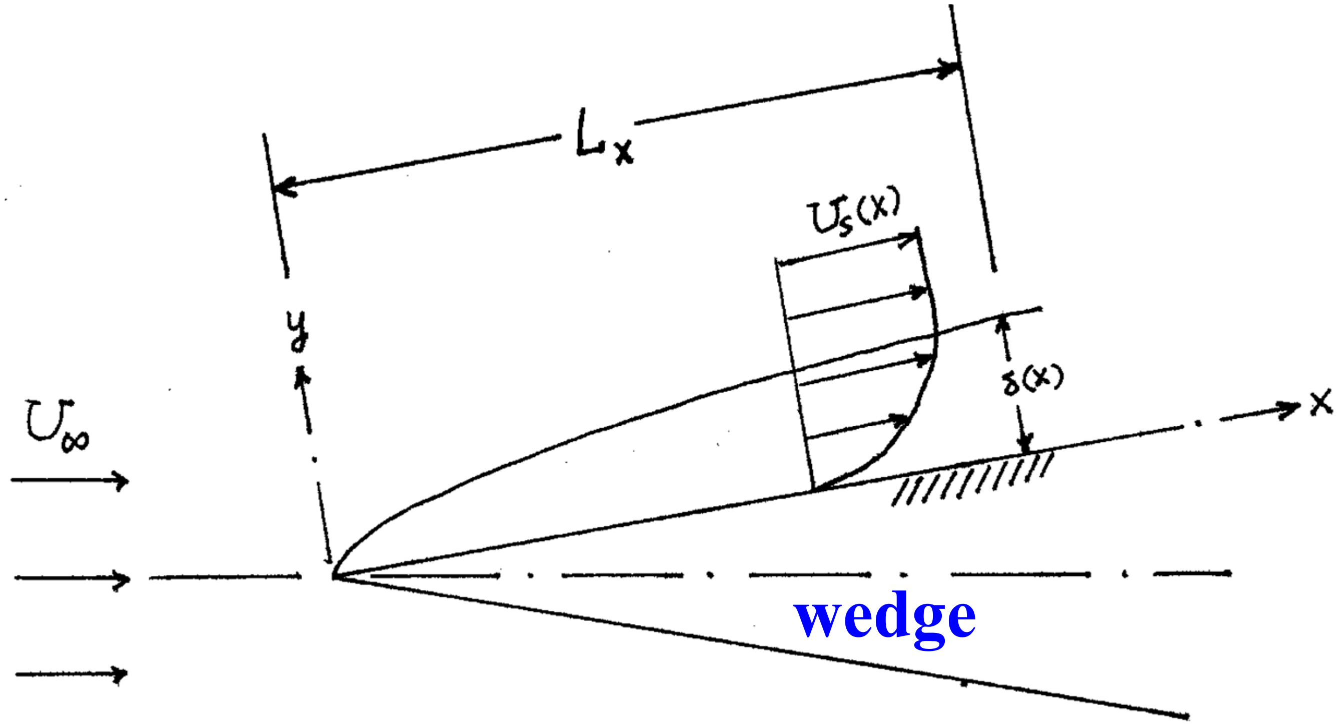
- The wake region sufficiently far downstream from the body (i.e., the **far wake**) can also be approximated as a thin boundary layer (without solid boundary), which is also governed by the boundary layer equations.

(Near wake - governed by the Continuity and Navier-Stokes equations)



Derivation of the boundary layer equation (1)

Consider the two-dimensional planar flow over a wedge at large Reynolds number.



$U_s(x)$: the velocity on the wedge surface of the inviscid flow

= the velocity at the outer edge of the boundary layer under **R** >> 1

Derivation of the boundary layer equation (2)

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \;\Rightarrow\; \frac{U_s}{L_x}\sim\frac{V_s}{\delta} \;\Rightarrow\; V_s\sim\frac{\delta}{L_x}U_s \quad (V_s \ll U_s \text{ as } \delta \ll L_x)$$

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{1}{\rho}\frac{\partial P}{\partial y}+\nu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)$$

$$U_s\frac{U_s\delta/L_x}{L_x}+(U_s\delta/L_x)\frac{U_s\delta/L_x}{\delta}\sim\frac{1}{\rho}\frac{\rho U_s^2}{\delta}+\nu\frac{U_s\delta/L_x}{L_x^2}+\nu\frac{U_s\delta/L_x}{\delta^2} \;\Rightarrow\; \frac{\delta^2}{L_x^2}+\frac{\delta^2}{L_x^2}\sim 1+\frac{1}{\mathbf{R}}\frac{\delta^2}{L_x^2}+\frac{1}{\mathbf{R}}$$

$$\Rightarrow\; \frac{\partial P}{\partial y}\approx 0 \quad \text{Then} \quad \frac{\partial P}{\partial x}=\frac{dP_s}{dx},$$

$P_s = P_s(x)$: surface pressure of the inviscid main flow

(the pressure gradient along the x-direction inside the boundary layer can be evaluated using the flow outside the boundary layer)

Derivation of the boundary layer equation (3)

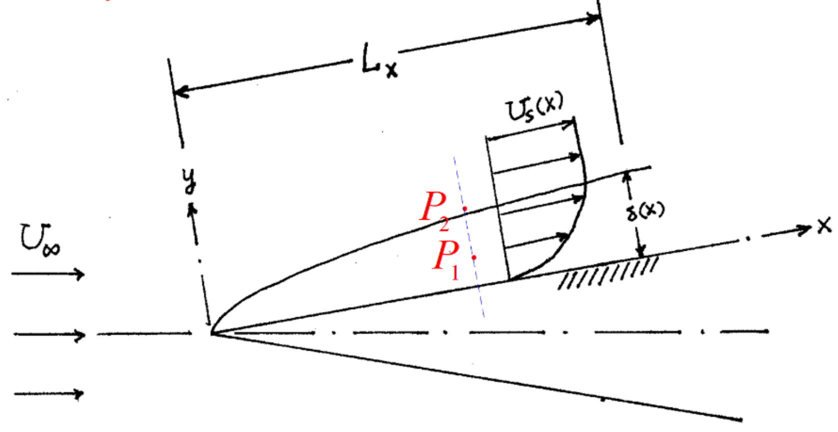
$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial P}{\partial x}+\nu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)$$

$$-\frac{1}{\rho}\frac{\partial P}{\partial x}=-\frac{1}{\rho}\frac{dP_s}{dx}=U_s\frac{dU_s}{dx}\leftarrow \rho\mathbf{u}\cdot\nabla\mathbf{u}=-\nabla P \text{ (Euler equation)}$$

three possibilities

$$U_s\frac{U_s}{L_x}+U_s\frac{\delta}{L_x}\frac{U_s}{\delta}\sim\frac{1}{\rho}\frac{\rho U_s^2}{L_x}+\nu\frac{U_s}{L_x^2}+\nu\frac{U_s}{\delta^2}\Rightarrow 1+1\sim 1+\frac{1}{\mathbf{R}}+\frac{1}{\mathbf{R}}\frac{L_x^2}{\delta^2}$$

$$\frac{\partial P}{\partial y}\approx 0\rightarrow P_2\approx P_1\approx P_s(x)\rightarrow\frac{\partial P}{\partial x}=\frac{dP_s}{dx}$$



$$\nu\frac{\partial^2 u}{\partial x^2}$$

(be neglected as R >>1)

Derivation of the boundary layer equation (4)

(i) If $(\delta / L_x)^2 \to 0$ slower than $1 / \mathrm{R} \to 0$ as $\mathrm{R} \to \infty$, the diffusion term $\left(\nu\, \partial^2 u / \partial y^2\right)$ is negligible and the streamwise Navier-Stokes equation reduces to the x-component of the Euler equation. The no-slip condition cannot be satisfied, and this is not a correct possibility.

(ii) If $(\delta / L_x)^2 \to 0$ faster than $1 / \mathrm{R} \to 0$ as $\mathrm{R} \to \infty$, the diffusion term $\left(\nu\, \partial^2 u / \partial y^2\right)$ dominates and the streamwise Navier-Stokes equation reduces to $\partial^2 u / \partial y^2 = 0$. Its solution subject to the no-slip condition is a linear profile, $u = cy$, which cannot be matched smoothly (the velocity is continuous but not its derivates) with the "external" inviscid main flow. Therefore, this is also not a correct possibility.

Derivation of the boundary layer equation (5)

(iii) If $(\delta / L_x)^2 \rightarrow 0$ is of the same order as $1 / \mathrm{R} \rightarrow 0$ as $\mathrm{R} \rightarrow \infty$, the cross-streamwise diffusion term $\left(\nu\, \partial^2 u / \partial y^2\right)$ is of the same order as the inertia and pressure term, and the streamwise Navier-Stokes equation reduces to

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\partial P}{\partial x} + \nu \frac{\partial^2 u}{\partial y^2}$$

Note that the above equation is a "**parabolic**" type partial differential equation, which is simpler than the "elliptic" Navier-Stokes equation.

Boundary layer equations over a planar wedge

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$$

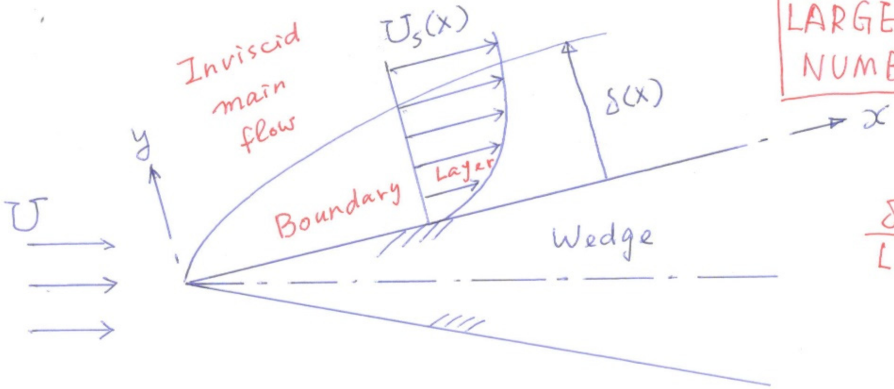
$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=U_s\frac{dU_s}{dx}+\nu\frac{\partial^2 u}{\partial y^2}$$

subject to $u=v=0$ at $y=0$

$u=U_s(x)$ as $y\to\infty$

and a suitable condition at a specified x.

LARGE REYNOLDS NUMBER, $R \to \infty$

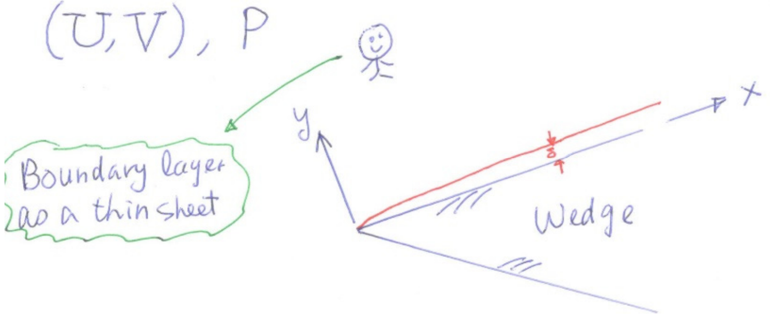


$$\frac{\delta}{L_x} \sim \frac{1}{\sqrt{R}} \to 0$$

Inviscid main flow

$L_x \sim L_y \gg \delta$

(U, V), P



$$\nabla \cdot \vec{U} = 0$$
$$\rho\, \vec{U} \cdot \nabla \vec{U} = -\nabla P$$

Slip BC's at $y=0$

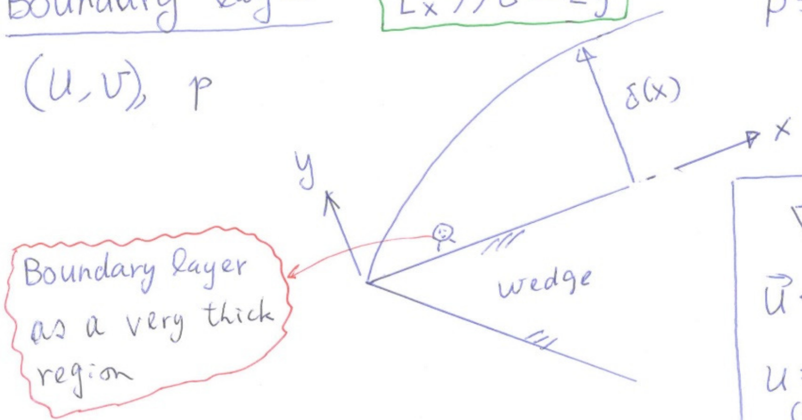
$U = U_s(x)$, $V = 0$ on $y=0$

$P = P_s(x)$

Boundary layer

$L_x \gg \delta = L_y$

(u, v), p



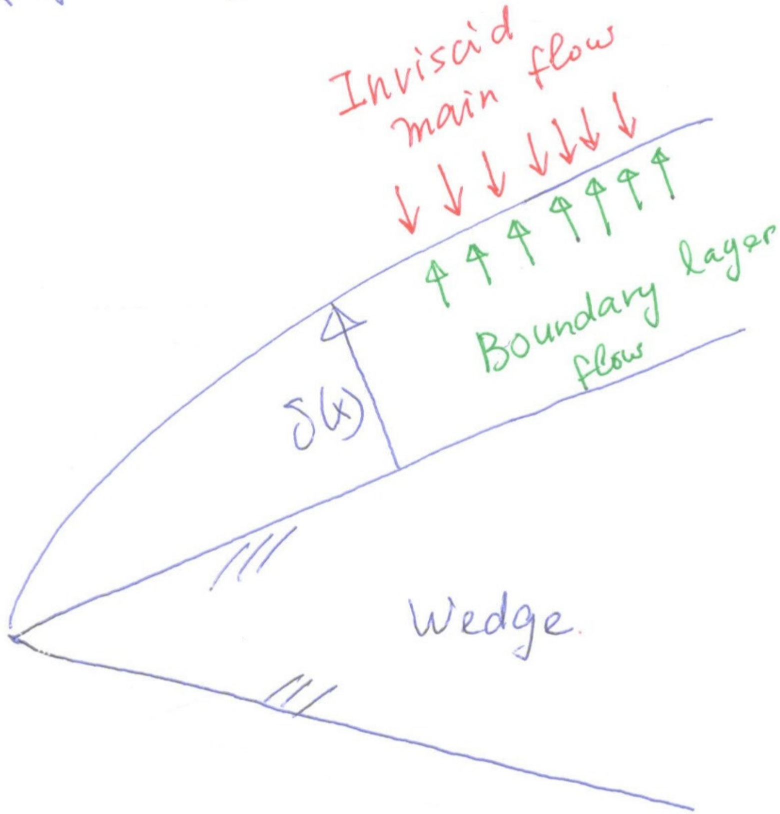
$$\nabla \cdot \vec{u} = 0$$
$$\vec{u} \cdot \nabla \vec{u} = U_s \frac{dU_s}{dx} + \nu \frac{\partial^2 u}{\partial y^2}$$

$u = v = 0$ on $y = 0$ (no-slip)

$u = U_s(x)$ as $y \to \infty$

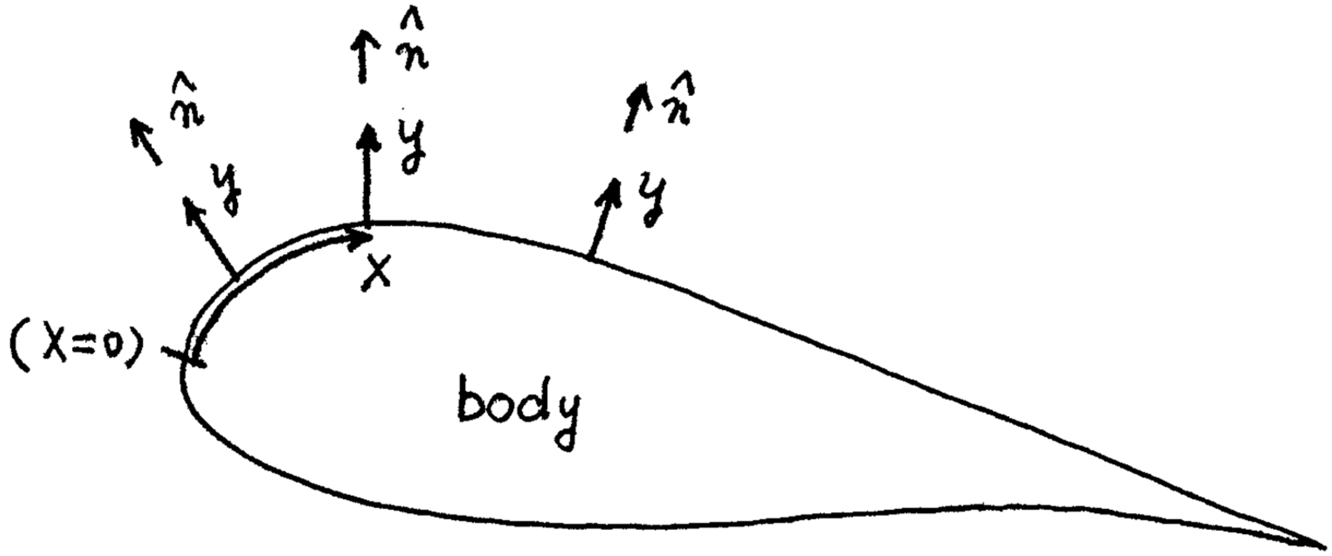
Matching (as $R \to \infty$, $\frac{\delta}{L_x} \sim \frac{1}{\sqrt{R}} \to 0$)

$$\lim_{\frac{y}{\delta} \to 0} \begin{pmatrix} \text{Inviscid} \\ \text{main} \\ \text{flow} \end{pmatrix} = \lim_{\frac{y}{\delta} \to \infty} \begin{pmatrix} \text{Boundary} \\ \text{layer} \\ \text{flow} \end{pmatrix}$$



Boundary layer equations over a planar curved surface

Under the boundary layer assumptions, we obtain the same equations as those in planar wedge for the boundary layer flow over a curved surface, provided that the boundary layer thickness is much less than the local radius of curvature of the body.



Unsteady planar boundary layer equations

For unsteady flow, the surface velocity of the inviscid flow is function of both x and t, i.e., $U_s(x,t)$.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{\partial U_s}{\partial t} + U_s\frac{\partial U_s}{\partial x} + \nu\frac{\partial^2 u}{\partial y^2}$$

- Subject to both the no-slip and matching boundary conditions
- Two dimensionless governing parameters: the **Reynolds number** and the **Strouhal number** (the ratio of the unsteady term to the convection term).

Unsteady and three-dimensional boundary layer flows

- For those who are interest in the problems of **unsteady** boundary layers, please read the books by Schlichting (Chapter 15) and by Rosenhead (Chapter 7).
- In a similar way, the boundary layer theory above can also be extended to boundary layer flows over **axial-symmetric** and **general three-dimensional** bodies (see Schlichting, Chapter 11 and Rosenhead, Chapter 8)

L. Rosenhead, "Laminar Boundary layer," Oxford University Press 1963.

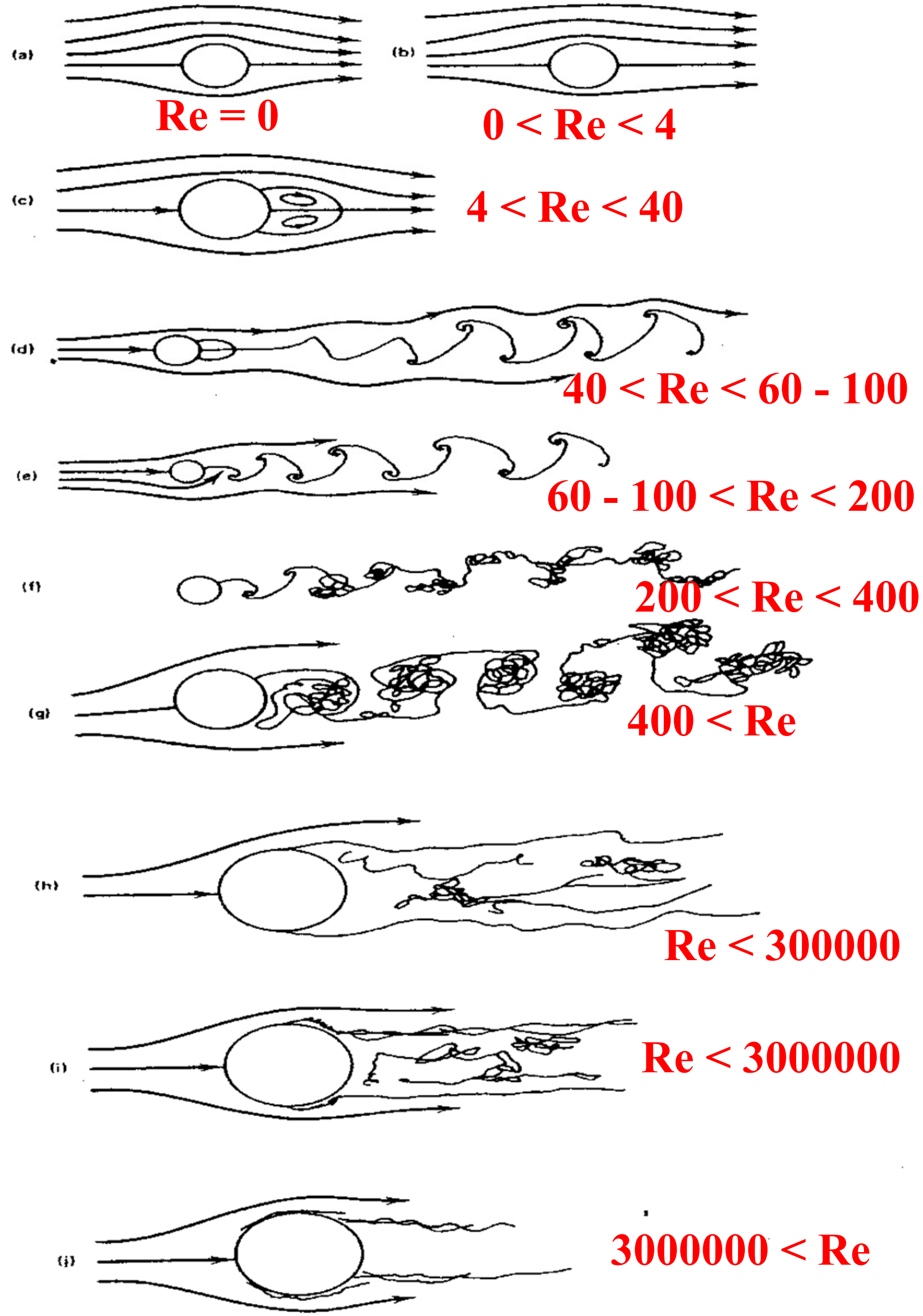
Cases for intermediate Reynolds numbers

When the Reynolds number, R, is of intermediate value, say, for R = O(1) – O(100), the inertia, the pressure and the viscous terms in the Navier-Stokes equation are of the same order, and the governing equations cannot be simplified. Note that the range for R = O(1) – O(100) corresponds to $\delta / L_s = O(1) - O(10)$. For moderate Reynolds number, the problem can be studied via **numerical method**

(see Chapter 14 of Panton's book, 2013 electronic version).

Flow over a circular cylinder – an example for illustrating the flow physics from small to large Reynolds number

- ➯ Detailed calculation procedures and results can be found from Chapter 14 of Panton's book.
- ➯ Essential flow physics can be summarized and sketched in the right Figure.
- ➯ It is interesting to note that the flow is unsteady at intermediate Reynolds number as the **Karman vortex street** sets on although the approaching flow is constant and the cylinder is fixed.



Flow regimes for a cylinder: (a) $Re = 0$, symmetrical; (b) $0 < Re < 4$; (c) $4 < Re < 40$, attached vortices; (d) $40 < Re < 60$–100, Kármán vortex street; (e) 60–$100 < Re < 200$, alternate shedding; (f) $200 < Re < 400$, vortices unstable to spanwise bending; (g) $400 < Re$, vortices turbulent at birth; (h) $Re < 3 \times 10^5$, laminar boundary layer separates at 80°; (i) $3 \times 10^5 < Re < 3 \times 10^6$, separated region becomes turbulent, reattaches, and separates again at 120°; (j) $3 \times 10^6 < Re$, turbulent boundary layer begins on front and separates on back.

Steady incompressible flow with constant viscosity

$\nabla \cdot \mathbf{u} = 0$ (Continuity equation)

$\rho \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla P + \mu \nabla^2 \mathbf{u}$ (Navier-Stokes equation) (2^{nd}-order)

10^{-5} 10^{-4} 10^{-3} 10^{-2} 10^{-1} 1 10^{1} 10^{2} 10^{3} 10^{4} 10^{5} $\mathbf{R} = \dfrac{\rho U_s L_s}{\mu}$

(Reynolds Number)

Low Reynolds number flows (R<<1) :

$\nabla \cdot \mathbf{u} = 0$

$0 \approx -\nabla p + \mu \nabla^2 \mathbf{u}$ (Stokes equation)

(2^{nd}-order)

Intermediate R (Numerical solution only - including the unsteady term)

High Reynolds number flows (R >> 1) :

$\nabla \cdot \mathbf{u} = 0$

$\rho \mathbf{u} \cdot \nabla \mathbf{u} \approx -\nabla p$ → **Inviscid main flow**

(Euler equation)

⇑

Matching (Inviscid main flow and Boundary layer)

⇓

$\nabla \cdot \mathbf{u} = 0$

$\rho \mathbf{u} \cdot \nabla \mathbf{u} \approx \rho U_s \dfrac{dU_s}{dx} + \mu \dfrac{\partial^2 \mathbf{u}}{\partial y^2}$ → **Boundary layer**

(Parabolized Navier-Stokes equation)

國立台灣大學 應用力學研究所
Institute of Applied Mechanics
National Taiwan University

Steady incompressible flow with constant viscosity

$\nabla \cdot \mathbf{u} = 0$ (Continuity equation)

$\rho \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla P + \mu \nabla^2 \mathbf{u}$ (Navier-Stokes equation) (2^{nd}-order)

10^{-5} 10^{-4} 10^{-3} 10^{-2} 10^{-1} 1 10^{1} 10^{2} 10^{3} 10^{4} 10^{5} $\mathbf{R} = \dfrac{\rho U_s L_s}{\mu}$

(Reynolds Number)

Low Reynolds number flows (R<<1) :

$\nabla \cdot \mathbf{u} = 0$

$0 \approx -\nabla p + \mu \nabla^2 \mathbf{u}$ (Stokes equation)

(2^{nd}-order)

Intermediate R (Numerical solution only - including the unsteady term)

High Reynolds number flows (R >> 1) :

$\nabla \cdot \mathbf{u} = 0$

$\rho \mathbf{u} \cdot \nabla \mathbf{u} \approx -\nabla p$ → **Inviscid main flow**

(Euler equation)

$\Uparrow$

Matching (Inviscid main flow and Boundary layer)

$\Downarrow$

$\nabla \cdot \mathbf{u} = 0$

$$\rho \mathbf{u} \cdot \nabla \mathbf{u} \approx \rho U_s \frac{dU_s}{dx} + \mu \frac{\partial^2 \mathbf{u}}{\partial y^2}$$ → **Boundary layer**

(Parabolized Navier-Stokes equation)