

# Graph Theory II (Math 7704)

## Spring 2024

### Lecturer

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NTU COOL site <https://cool.ntu.edu.tw/courses/35543>

**Topics of the course** The fundamental extremal problem asks how large a structure can be without containing a forbidden substructure. This very flexible problem arises in many different settings and enjoys several applications. A variety of methods have been developed to tackle them, and this course will pay special attention to the probabilistic, linear algebraic and (briefly) topological methods.

The course will be divided into three sections addressing the following topics:

#### 1. Extremal graph theory and the probabilistic method

- Ramsey Theory
- Turán Theory
- The Regularity Lemma
- Roth's Theorem

#### 2. Extremal combinatorics and the linear algebraic method

- Sperner's Theorem
- Kruskal–Katona Theorem
- Erdős–Ko–Rado Theorem
- Restricted intersections

#### 3. Topological methods

- Sperner's Lemma
- Independent transversals
- Kneser's conjecture

**Literature** The course will be self-contained, and course notes will be available on the COOL site. For additional reading, students may wish to consult the following texts.

- N. Alon and J. Spencer: The Probabilistic Method
- L. Babai and P. Frankl: Linear Algebra Methods in Combinatorics
- R. Diestel: Graph Theory
- J. Fox: Lecture notes

- S. Jukna: Extremal Combinatorics
- J. Matoušek: Using the Borsuk-Ulam Theorem
- J. van Lint and R. Wilson: A Course in Combinatorics
- D. West: Introduction to Graph Theory

**Prerequisites** Successful completion of Graph Theory I, or an equivalent course. Familiarity with linear algebra and (discrete) probability is necessary, and some knowledge of topology would be a bonus, but is not required.

**Schedule** There will be two lectures each week, on Tuesdays from 15:30 to 17:20, and on Fridays from 12:20 to 13:10. The lectures will be held in Room 102 of the Astro-Math building. Office hours will be available upon request - please ask in class or send me an e-mail to set one up. You should also feel free to post questions in the Discussions on NTU COOL.

**Grading** The final grades for the course will be weighted as follows:

- Homework — 30%
- Midterm exam — 30%
- Final exam — 40%

**Homework** There will be homework assignments every two weeks, covering the topics introducing in the preceding lectures. The assignments will be available on the COOL site, and you should submit your solutions there as well.

**Exams** There will be a midterm exam and a final exam for the course, which will be scheduled closer to the date. These exams will be closed-book; you are not allowed to use any notes or texts while writing the exam. On the exam, you will be faced with two kinds of tasks:

- Lexical knowledge: definitions, statements and proofs of theorems from lectures
- Problem solving: applying the theorems and methods to solve exercises (these will be similar in nature to what you encounter on the homework)