CHAPTER 25
Credit Derivatives

Practice Questions

25.1
Both provide insurance against a particular company defaulting during a period of time. In a credit default swap, the payoff is the notional principal amount multiplied by one minus the recovery rate. In a binary swap, the payoff is the notional principal.

25.2
The seller receives 300,000,000×0.0060×0.25 = $450,000 at times 0.25, 0.50, 0.75, 1.00,…. 4.0 years. The seller also receives a final accrual payment of about $300,000 (=$300,000,000×0.0060× 2/12) at the time of the default (4 years and two months). The seller pays

$$300,000,000 \times 0.6 = \$180,000,000$$

at the time of the default. (This does not consider day count conventions.)

25.3
A cash CDO is created by buying bonds and tranching out the risks. A synthetic CDO is created from a portfolio of short CDSs (i.e., CDS that are selling protection).

25.4
Single tranche trading occurs when a tranche of a synthetic CDO is traded without the underlying portfolios of short CDSs being created. The underlying portfolio is simply used as a reference point to determine cash flows on the tranche.

25.5
In a first-to-default basket CDS there are a number of reference entities. When the first one defaults, there is a payoff (calculated in the usual way for a CDS) and basket CDS terminates. The value of a first-to-default basket CDS decreases as the correlation between the reference entities in the basket increases. This is because the probability of a default is high when the correlation is zero and decreases as the correlation increases. In the limit when the correlation is one, there is in effect only one company and the probability of a default is quite low.

25.6
Risk-neutral default probabilities are backed out from credit default swaps or bond prices. Real-world default probabilities are calculated from historical data. Risk-neutral probabilities should be used for valuation (e.g., the valuations of CDSs). Real world probabilities should be used for scenario analysis.

25.7
Suppose a company wants to buy some assets. If a total return swap is used, a financial institution buys the assets and enters into a swap with the company where it pays the company the return on the assets and receives from the company LIBOR plus a spread. The financial institution has less risk than it would have if it lent the company money and used the assets as collateral. This is because, in the event of a default by the company, it owns the

assets.

25.8
The table corresponding to Tables 25.1, giving unconditional default probabilities, is as follows:

Time (years)	*Probability of surviving to year end*	*Default Probability during year*
1	0.9704	0.0296
2	0.9418	0.0287
3	0.9139	0.0278
4	0.8869	0.0270
5	0.8607	0.0262

The table corresponding to Table 25.2, giving the present value of the expected regular payments (payment rate is s per year), is as follows:

Time (yrs)	*Probability of survival*	*Expected Payment*	*Discount Factor*	*PV of Expected Payment*
1	0.9704	0.9704s	0.9324	0.9048s
2	0.9418	0.9418s	0.8694	0.8187s
3	0.9139	0.9139s	0.8106	0.7408s
4	0.8869	0.8869s	0.7558	0.6703s
5	0.8607	0.8607s	0.7047	0.6065s
Total				3.7412s

The table corresponding to Table 25.3, giving the present value of the expected payoffs (notional principal =$1), is as follows:

Time (yrs)	*Probability of default*	*Recovery Rate*	*Expected Payoff*	*Discount Factor*	*PV of Expected Payment*
0.5	0.0296	0.3	0.0207	0.9656	0.0200
1.5	0.0287	0.3	0.0201	0.9003	0.0181
2.5	0.0278	0.3	0.0195	0.8395	0.0164
3.5	0.0270	0.3	0.0189	0.7827	0.0148
4.5	0.0262	0.3	0.0183	0.7298	0.0134
Total					0.0826

The table corresponding to Table 25.4, giving the present value of accrual payments, is as follows:

Time (yrs)	*Probability of default*	*Expected Accrual Payment*	*Discount Factor*	*PV of Expected Accrual Payment*
0.5	0.0296	0.0148*s*	0.9656	0.0143s
1.5	0.0287	0.0143*s*	0.9003	0.0129s
2.5	0.0278	0.0139*s*	0.8395	0.0117s
3.5	0.0270	0.0135*s*	0.7827	0.0106s
4.5	0.0262	0.0131*s*	0.7298	0.0096s
Total				0.0590s

The credit default swap spread s is given by:

$$3.7412s+0.0590s = 0.0826$$

It is 0.0217 or 217 basis points. This can be verified with DerivaGem.

25.9

If the credit default swap spread is 150 basis points, the value of the swap to the buyer of protection is:

$$0.0826 - (3.7412 + 0.0590) \times 0.0150 = 0.0256$$

per dollar of notional principal.

25.10

If the swap is a binary CDS, the present value of expected payoffs is calculated as follows:

Time (years)	*Probability of Default*	*Expected Payoff*	*Discount Factor*	*PV of expected Payoff*
0.5	0.0296	0.0296	0.9656	0.0285
1.5	0.0287	0.0287	0.9003	0.0258
2.5	0.0278	0.0278	0.8395	0.0234
3.5	0.0270	0.0270	0.7827	0.0211
4.5	0.0262	0.0262	0.7298	0.0191
				0.1180

The credit default swap spread s is given by:

$$3.7412s + 0.0590s = 0.1180$$

It is 0.0310 or 310 basis points.

25.11

A five-year n th to default credit default swap works in the same way as a regular credit default swap except that there is a basket of companies. The payoff occurs when the n th default from the companies in the basket occurs. After the n th default has occurred, the swap ceases to exist. When $n = 1$ (so that the swap is a "first to default"), an increase in the default correlation lowers the value of the swap. When the default correlation is zero, there are 100 independent events that can lead to a payoff. As the correlation increases, the probability of a payoff decreases. In the limit when the correlation is perfect, there is in effect only one company and therefore only one event that can lead to a payoff.

When $n = 25$ (so that the swap is a 25th to default), an increase in the default correlation increases the value of the swap. When the default correlation is zero, there is virtually no chance that there will be 25 defaults and the value of the swap is very close to zero. As the correlation increases, the probability of multiple defaults increases. In the limit when the correlation is perfect, there is in effect only one company and the value of a 25th-to-default credit default swap is the same as the value of a first-to-default swap.

25.12.
The payoff is $L(1-R)$ where L is the notional principal and R is the recovery rate.

25.13.
The payoff from a plain vanilla CDS is $1-R$ times the payoff from a binary CDS with the same principal. The payoff always occurs at the same time on the two instruments. It follows that the regular payments on a new plain vanilla CDS must be $1-R$ times the payments on a new binary CDS. Otherwise, there would be an arbitrage opportunity.

25.14
The 1.63% hazard rate can be calculated by setting up a worksheet in Excel and using Solver. To verify that 1.63% is correct we note that, with a hazard rate of 1.63%, the table is as follows:

Time (years)	*Probability of surviving to year end*	*Default Probability during year*
1	0.9838	0.0162
2	0.9679	0.0159
3	0.9523	0.0156
4	0.9369	0.0154
5	0.9217	0.0151

The present value of the regular payments becomes 4.1162s, the present value of the expected payoffs becomes 0.0416, and the present value of the expected accrual payments becomes 0.0347s.When s = 0.01, the present value of the expected payments equals the present value of the expected payoffs.
When the recovery rate is 20%, the implied hazard rate (calculated using Solver) is 1.22% per year. Note that 1.22/1.63 is approximately equal to $(1-0.4)/(1-0.2)$ showing that the implied hazard is approximately proportional to $1/(1-R)$.
In passing we note that if the CDS spread is used to imply an unconditional default probability (assumed to be the same each year) then this implied unconditional default probability is exactly proportional to $1/(1-R)$. When we use the CDS spread to imply a hazard rate (assumed to be the same each year) it is only approximately proportional to $1/(1-R)$.

25.15
In the case of a total return swap, a company receives (pays) the increase (decrease) in the value of the bond. In the regular swap, this does not happen.

25.16
When a company enters into a long (short) forward contract, it is obligated to buy (sell) the protection given by a specified credit default swap with a specified spread at a specified future time. When a company buys a call (put) option contract, it has the option to buy (sell) the protection given by a specified credit default swap with a specified spread at a specified

future time. Both contracts are normally structured so that they cease to exist if a default occurs during the life of the contract.

25.17
A credit default swap insures a corporate bond issued by the reference entity against default. Its approximate effect is to convert the corporate bond into a risk-free bond. The buyer of a credit default swap has therefore chosen to exchange a corporate bond for a risk-free bond. This means that the buyer is long a risk-free bond and short a similar corporate bond.

25.18
Payoffs from credit default swaps depend on whether a particular company defaults. Arguably, some market participants have more information about this than other market participants. (See Business Snapshot 25.2.)

25.19
Real world default probabilities are less than risk-neutral default probabilities. It follows that the use of real world (historical) default probabilities will tend to understate the value of a CDS.

25.20
In an asset swap, the bond's promised payments are swapped for floating reference rate plus a spread. In a total return swap, the bond's actual payments are swapped for floating reference rate plus a spread.

25.21
Using equation (25.5), the probability of default conditional on a factor value of F is

$$N\left(\frac{N^{-1}(0.03)-\sqrt{0.2}F}{\sqrt{1-0.2}}\right)$$

For F equal to –2, –1, 0, 1, and 2 the probabilities of default are 0.135, 0.054, 0.018, 0.005, and 0.001 respectively. To six decimal places, the probability of more than 10 defaults for these values of F can be calculated using the BINOMDIST function in Excel. They are 0.959284, 0.079851, 0.000016, 0, and 0, respectively.

25.22
Compound correlation for a tranche is the correlation which when substituted into the one-factor Gaussian copula model produces the market quote for the tranche. Base correlation is the correlation which is consistent with the one-factor Gaussian copula and market quotes for the 0 to X% tranche where X% is a detachment point. It ensures that the expected loss on the 0 to X% tranche equals the sum of the expected losses on the underlying traded tranches.

25.23
In this case, $a_L = 0.09$ and $a_H = 0.12$. Proceeding similarly in Example 25.2, the tranche spread is calculated as 30 basis points.

25.24
The table corresponding to Table 25.2, giving the present value of the expected regular payments (payment rate is s per year), is as follows:

Time (yrs)	*Probability of survival*	*Expected Payment*	*Discount Factor*	*PV of Expected Payment*
0.5	0.990	0.4950s	0.9704	0.4804s
1.0	0.980	0.4900s	0.9418	0.4615s
1.5	0.965	0.4825s	0.9139	0.4410s
2.0	0.950	0.4750s	0.8869	0.4213s
Total				1.8041s

The table corresponding to Table 25.3, giving the present value of the expected payoffs (notional principal =$1), is as follows:

Time (yrs)	*Probability of default*	*Recovery Rate*	*Expected Payoff*	*Discount Factor*	*PV of Expected Payment*
0.25	0.010	0.2	0.008	0.9851	0.0079
0.75	0.010	0.2	0.008	0.9560	0.0076
1.25	0.015	0.2	0.012	0.9277	0.0111
1.75	0.015	0.2	0.012	0.9003	0.0108
Total					0.0375

The table corresponding to Table 25.4, giving the present value of accrual payments, is as follows:

Time (yrs)	*Probability of default*	*Expected Accrual Payment*	*Discount Factor*	*PV of Expected Accrual Payment*
0.25	0.010	0.0025s	0.9851	0.0025s
0.75	0.010	0.0025s	0.9560	0.0024s
1.25	0.015	0.00375s	0.9277	0.0035s
1.75	0.015	0.00375s	0.9003	0.0034s
Total				0.0117s

The credit default swap spread s is given by:

$$1.804s + 0.0117s = 0.0375$$

It is 0.0206 or 206 basis points. For a binary credit default swap, we set the recovery rate equal to zero in the second table to get the present value of expected payoffs equal to 0.0468 so that

$$1.804s + 0.0117s = 0.0468$$

and the spread is 0.0258 or 258 basis points.

25.25

The spread for a binary credit default swap is equal to the spread for a regular credit default swap divided by $1-R$ where R is the recovery rate. This means that $1-R$ equals 0.75 so that the recovery rate is 25%. To find λ, we search for the conditional annual default rate that leads to the present value of payments being equal to the present value of payoffs. The answer is λ = 0.0156. The present value of payoffs (per dollar of principal) is then 0.0499. The present value of regular payments is 4.1245. The present value of accrual payments is 0.0332.

25.26

As the correlation increases, the yield on the equity tranche decreases and the yield on the senior tranches increases. To understand this, consider what happens as the correlation increases from zero to one. Initially, the equity tranche is much more risky than the senior tranche. But as the correlation approaches one, the companies become essentially the same. We are then in the position where either all companies default or no companies default and the tranches have similar risk.

25.27

When the credit default swap spread is 150 basis points, an arbitrageur can earn more than the risk-free rate by buying the corporate bond and buying protection. If the arbitrageur can finance trades at the risk-free rate (by shorting the riskless bond), it is possible to lock in an almost certain profit of 100 basis points. When the credit spread is 300 basis points, the arbitrageur can short the corporate bond, sell protection and buy a risk free bond. This will lock in an almost certain profit of 50 basis points. The arbitrage is not perfect for a number of reasons:

(a) It assumes that both the corporate bond and the riskless bond are par yield bonds and that interest rates are constant. In practice, the riskless bond may be worth more or less than par at the time of a default so that a credit default swap under protects or overprotects the bond holder relative to the position with a riskless bond.

(b) There is uncertainty created by the cheapest-to-delivery bond option.

(c) To be a perfect hedge, the credit default swap would have to give the buyer of protection the right to sell the bond for face value plus accrued interest, not just face value.

The arbitrage opportunities assume that market participants can short corporate bonds and borrow at the risk-free rate. The definition of the credit event in the ISDA agreement is also occasionally a problem. It can occasionally happen that there is a "credit event" but promised payments on the bond are made.

25.28

(a) In this case, the answer to Example 25.3 gets modified as follows. When $F = -1.0104$, the cumulative probabilities of one or more defaults in 1, 2, 3, 4, and 5 years are 0.3075, 0.5397, 0.6959, 0.7994, and 0.8676. The conditional probability that the first default occurs in years 1, 2, 3, 4, and 5 are 0.3075, 0.2322, 0.1563, 0.1035, and 0.0682, respectively. The present values of payoffs, regular payments, and accrual payments conditional on $F = -1.0104$ are 0.4767, 1.6044*s*, and 0.3973*s*. Similar calculations are carried out for the other factor values. The unconditional expected present values of payoffs, regular payments, and accrual payments are 0.2602, 2.9325*s*, and 0.2168*s*. The breakeven spread is therefore

$$0.2602/(2.9325 + 0.2168) = 0.0826$$

or 826 basis points.

(b) In this case, the answer to Example 25.3 gets modified as follows. When $F = -1.0104$, the cumulative probabilities of two or more defaults in 1, 2, 3, 4, and 5 years are 0.0483, 0.1683, 0.3115, 0.4498, and 0.5709. The conditional probability that the second default occurs in years 1, 2, 3, 4, and 5 are 0.0483, 0.1200, 0.1432, 0.1383, and 0.1211, respectively. The present values of payoffs, regular payments, and accrual payments conditional on $F = -1.0104$ are 0.2986, 3.0351*s*, and 0.2488*s*. Similar calculations are carried out for the other factor values. The unconditional expected present values of payoffs, regular payments, and accrual payments are 0.1264, 3.7428*s*, and 0.1053*s*. The breakeven spread is therefore

$$0.1264/(3.7428 + 0.1053) = 0.0328$$

or 328 basis points.

25.29

In this case, $a_L = 0.06$ and $a_H = 0.09$. Proceeding similarly in Example 25.2, the tranche spread is calculated as 98 basis points assuming a tranche correlation of 0.15.

25.30 (Excel file)

The hazard rate consistent with the data is 1.28%. The compound (tranche) correlations are 0.4017, 0.8425, 0.1136, 0.2198, and 0.3342. The base correlations are 0.4017, 0.5214, 0.5825, 0.6046, and 0.7313. Note that in January 2009 spreads were so high that correlations could not be implied and many dealers had to change their models.