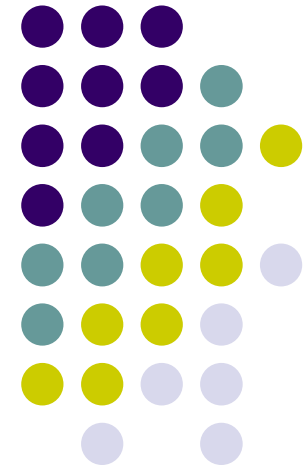


Exponential and logarithmic functions (6)

Applications





- Population model (logistic)
- Cooling and carbon dating
- Gaussian distribution
- Logarithmic scale

Population model (logistic)



- Exponential Growth

- Growth rate proportional to own quantity:

Bacterial colony, spread of virus, debt with interest

- **Example:** model the population if the community had 10000 in 1980 and 200000 in 2000.

$$P(t) = P_0 \cdot e^{kt} \text{ with } P(0) = 1, P(20) = 20$$

Population model (logistic)



- Logistic Models

- Given Carrying capacity K , $P(t) = \frac{K}{1+Ke^{-rt}}$, $\lim_{t \rightarrow \infty} P(t) = K$
- Standard logistic function: $f(x) = \frac{1}{1+e^{-x}}$
- **Example:** If $f(x)$ represents percentage to max capacity, when will we reach 80% ? 99.9%?

Cooling and carbon dating



- Newton's Law of Cooling

$$T(t) = K + (T(0) - K) \cdot e^{-rt}$$

Temperature at time t Room temperature

$$\lim_{t \rightarrow \infty} e^{-rt} = 0$$

$$\lim_{t \rightarrow \infty} T(t) = K$$

- Carbon Dating (C-14 half-life ~5730 years)

$$C(t) = C_0 \cdot \left(\frac{1}{2}\right)^{\frac{t}{5730}}$$

When $t = 5730$: $\frac{C(t)}{C_0} = \frac{1}{2}$

$$C(t) = 12.5\% C_0 \Rightarrow t = 3 \times 5730 = 17190$$

Gaussian Distribution



- Standard Normal Distribution: $\varphi(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}$
- Mean = 0, standard deviation = 1
- 68 – 95 – 99.7 Rule
- Example: Intelligence Quotient has $\mu = 100, \sigma = 15$
where $\mu = \text{average}(\text{mean}), \sigma = \text{standard deviation}$

Logarithmic scale



- Richter magnitude, decibel, music, acidity
- Converts exponential growth to linear growth

Example

- $y = K \cdot e^{rx}, Y = \ln y = \ln K + r \cdot x$
- Order of magnitude
- Octave $\log_2\left(\frac{f_2}{f_1}\right)$, compare frequencies
- pH: $\log\left(\frac{1}{a_{H^+}}\right)$, hydrogen ion activity

Logarithmic scale



- Log Plot

- Log-log plots $y = f(x) \rightarrow Y = \ln f(e^x)$
- **Example: Cobb-Douglas production**
- Semi-log plots $y = f(x) \rightarrow Y = \ln f(x)$
- **Example: Progression of pandemic**



- What are some different population models?
- How does exponential decay show up in real life?
- What is a normal distribution?
- Where would you see logarithmic scale in action?