Polynomials (2)

Equations of Lines

Outline

- Equations of Lines
  - The Point-Slope Form (點斜式)
  - The Two-Point Form (兩點式)
  - The Point-Normal Form (點法式)
- Orthogonal (垂直) Lines

數學系 蔡雅如老師

The Point-Slope Form

If the **slope**, m, and a **point** on the line, (x_0, y_0), are given, then the line is uniquely determined. Moreover, the equation of the line is

$$y = y_0 + m(x - x_0)$$

which is called the **point-slope form** of the line.

Example 2.1.

Find the equation of the line with slope 3 passing the point (2,1).

The Two-Point Form

A line can also be uniquely determined by given any **two points** on it, say, (x_1, y_1) and (x_2, y_2). The equation of the line is :

$$y = y_1 + \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

Which is called the **two-point form** of the line.

Example 2.2.

Find the equation of the line passing points (2,0) and (1,4).

- The Two-Point Form

Example 2.3.

Find the equation of the line passing points $(-2,1)$ and $(-2,3)$.

Warning The line containing points (x_1, y_1) and (x_1, y_2) is **"vertical"** with fixed x coordinate. Hence its equation is

$$x = x_1.$$

It is impossible to compute the slope of a vertical line (since for any two points on the line, we have $\Delta x = 0$), and we say that its **slope is "infinity"** which is denoted by ∞.

• The Point-Normal Form

Sometimes we are given a normal vector, $\boldsymbol{n} = (\boldsymbol{a}, \boldsymbol{b})$, and a point, $(\boldsymbol{x_0}, \boldsymbol{y_0})$, of a line. Then, the equation of the line is

$$a(x - x_0) + b(y - y_0) = 0$$

which is called the **point-normal form** of the line.

Example 2.4.

Find the equation of the line with normal vector $\boldsymbol{n} = (1, -2)$ passing the point (2,3).

• Orthogonal Lines

At the end of this section, we discuss the relationship between slopes of two **orthogonal (垂直的) lines**.

The following proposition gives us an algebraic method to determine whether two lines are orthogonal.

Proposition 2.4.

Lines $y = m_1 x + b_1$ and $y = m_2 x + b_2$ are orthogonal if and only if

$$m_1 \cdot m_1 = -1.$$

- Orthogonal Lines

Example 2.3.

Find the equation of the line that is orthogonal to the line $2x - 3y = 6$ and passes the point $(-2,1)$.

Review

- What is the point-slope form of a line?
- What is the two-point form of a line?
- What is the point-normal form of a line?
- How do we determine whether two lines are orthogonal?

數學系 蔡雅如老師