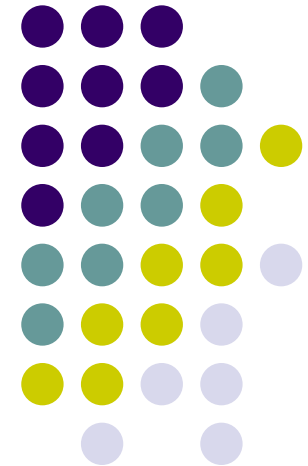


# Polynomials (2)

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Equations of Lines





- Equations of Lines
  - The Point-Slope Form (點斜式)
  - The Two-Point Form (兩點式)
  - The Point-Normal Form (點法式)
- Orthogonal (垂直) Lines

# Equations of Lines



- The Point-Slope Form

If the **slope,  $m$** , and a **point** on the line,  **$(x_0, y_0)$** , are given, then the line is uniquely determined. Moreover, the equation of the line is

$$y = y_0 + m(x - x_0)$$

which is called the **point-slope form** of the line.

## Example 2.1.

Find the equation of the line with slope 3 passing the point (2,1).



- The Two-Point Form

A line can also be uniquely determined by given any **two points** on it, say,  $(x_1, y_1)$  and  $(x_2, y_2)$ . The equation of the line is :

$$y = y_1 + \frac{y_2 - y_1}{x_2 - x_1} (x - x_1)$$

Which is called the **two-point form** of the line.

## Example 2.2.

Find the equation of the line passing points  $(2,0)$  and  $(1,4)$ .

# Equations of Lines



## • The Two-Point Form

### Example 2.3.

Find the equation of the line passing points  $(-2,1)$  and  $(-2,3)$ .

**Warning** The line containing points  $(x_1, y_1)$  and  $(x_1, y_2)$  is “**vertical**” with fixed  $x$  coordinate. Hence its equation is

$$x = x_1.$$

It is impossible to compute the slope of a vertical line (since for any two points on the line, we have  $\Delta x = 0$ ), and we say that its **slope is “infinity”** which is denoted by  $\infty$ .

# Equations of Lines



- The Point-Normal Form

Sometimes we are given a normal vector,  $\mathbf{n} = (a, b)$ , and a point,  $(x_0, y_0)$ , of a line. Then, the equation of the line is

$$a(x - x_0) + b(y - y_0) = 0$$

which is called the **point-normal form** of the line.

## Example 2.4.

Find the equation of the line with normal vector  $\mathbf{n} = (1, -2)$  passing the point  $(2, 3)$ .



- Orthogonal Lines

At the end of this section, we discuss the relationship between slopes of two **orthogonal (垂直的) lines**.

The following proposition gives us an algebraic method to determine whether two lines are orthogonal.

**Proposition 2.4.**

Lines  $y = m_1x + b_1$  and  $y = m_2x + b_2$  are orthogonal if and only if

$$m_1 \cdot m_2 = -1.$$



- Orthogonal Lines

## Example 2.3.

Find the equation of the line that is orthogonal to the line  $2x - 3y = 6$  and passes the point  $(-2, 1)$ .



# Review



- What is the point-slope form of a line?
- What is the two-point form of a line?
- What is the point-normal form of a line?
- How do we determine whether two lines are orthogonal?